Developing time implicit solvers for stellar astrophysics: The MUSIC code

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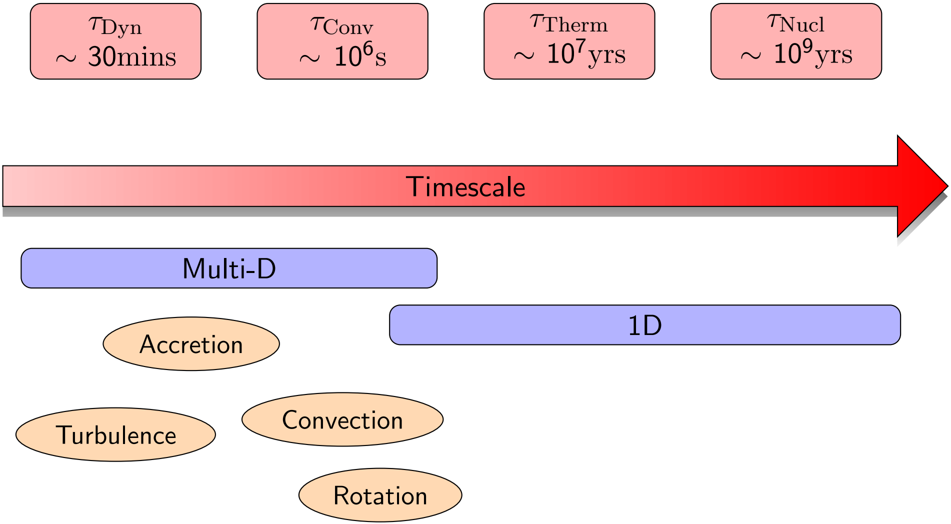
Motivation


τ_{Dyn}
$\sim$ 30mins
τ_{Conv}
$\sim 10^6$s
τ_{Therm}
$\sim 10^7$yrs
τ_{Nucl}
$\sim 10^9$yrs
Timescale
Multi-D
1D
Accretion
Turbulence
Convection
Rotation

Code Specification

Some code requirements:

- Include same (micro-)physics as 1D stellar evolution methods
  - Tabulated EOS (incl. ionisation, partial degeneracy) & opacity (OPAL)
- Work at or close to HSE
  - Staggered grid to simplify balance
- Arbitrary stratification & $\mathrm{M_s} \lesssim 1$
  - Fully compressible hydro
- Also low Mach requirements
  - Simultaneously model $10^{-6} \leq \mathrm{M_s} < 1$
- Global stellar models
  - Spherical coordinates (but also option for Cartesian)

The Equations

$$\frac{\partial}{\partial t}\rho = -\nabla \cdot (\rho \vec{u}) = R_U^{\rho}$$

$$\frac{\partial}{\partial t}\rho e = -\nabla \cdot (\rho e \vec{u}) - P \nabla \cdot \vec{u} - \nabla \cdot (\chi \nabla T) = R_U^{\rho \epsilon}$$

$$\frac{\partial}{\partial t}\rho \vec{u} = -\nabla \cdot (\rho \vec{u} \otimes \vec{u}) - \nabla P + \rho \vec{g} = R_U^{\rho \vec{u}}$$

More compactly:

$$\frac{dU}{dt} = R_U(X), \quad U = (\rho, \rho\epsilon, \rho\vec{u}) \quad X = (\rho, \epsilon, \vec{u})$$

Time implicit:

$$U\left(X^{n+1}\right) = U\left(X^{n}\right) + \frac{\Delta t}{2}\left(R_U\left(X^{n}\right) + R_U\left(X^{n+1}\right)\right)$$

Iteratively solve non-linear method where each correction is defined by: $\mathbf{J}\delta\vec{X} = -F_U(X)$.

Viallet et al 2016. Astronomy & Astrophysics, 586.

Solution Method

Linear problem solved using preconditioned GMRES:

$$\left(\mathbf{JM}^{-1}\right)\delta\vec{X}' = -F_U(X),$$

$$\delta X = \mathbf{M}^{-1}\delta X', \quad \text{where} \quad \mathbf{M}^{-1} \approx \mathbf{J}^{-1}.$$

Explicit Jacobian

- Explicit formation of Jacobian (using CFD). Memory intensive.
- "Black box" preconditioner (e.g. ILU), but performance poor for $\mathrm{CFL} > 100$
- Low Mach can be improved using block-Jacobi left prec. Effective Jacobian size doubles.

Jacobian Free

- Approximate matrix-vector product using finite difference. **Low memory**
- Requires physics-specific semi-implicit(SI), preconditioning methods. Extra development.
- Used within **fully implicit method** efficient upto $\mathrm{CFL} \approx 10^5$

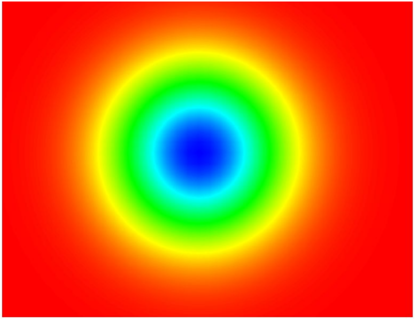
Isentropic Vortex

Problem Definition

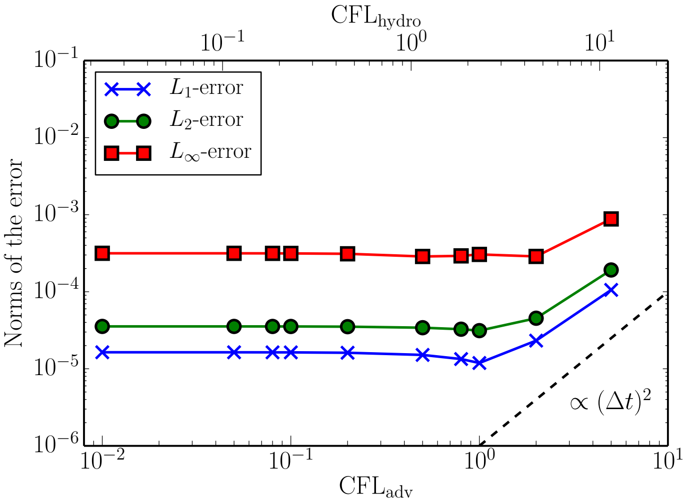
$$\rho = (\mathrm{T}_0 + \delta \mathrm{T})^{\frac{1}{\gamma-1}} \quad e = \frac{\rho^{\gamma-1}}{\gamma - 1}$$

$$u = 1.0 + \delta u \quad v = \delta v$$

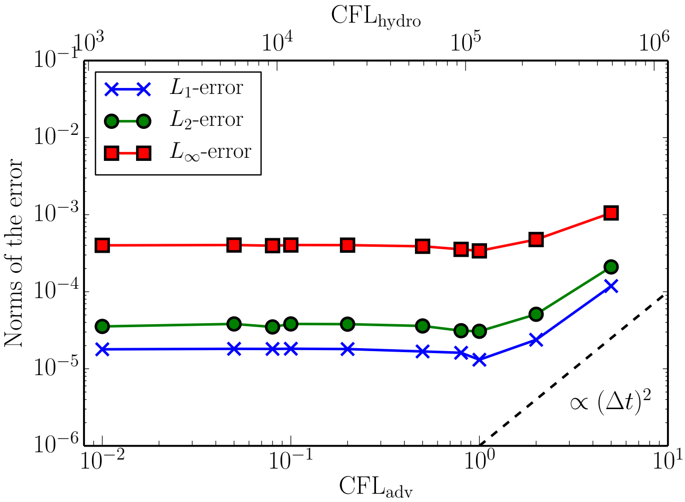
- Vary M_s by varying T_0
- Advect vortex until $\mathrm{t} = 0.4$ for various time-steps.
- Investigate how error behaviour varies with Mach number.



Isentropic Vortex: $M_s = 10^{-1}$



Isentropic Vortex: $M_s = 10^{-6}$



Taylor Green Vortex

Problem Definition

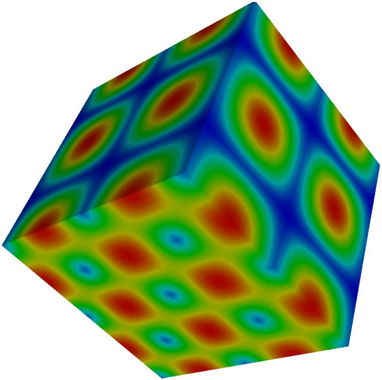
$$\rho = 1.0 \; p = p_0 + \delta P$$

$$u_x = \sin x \cos y \cos z$$

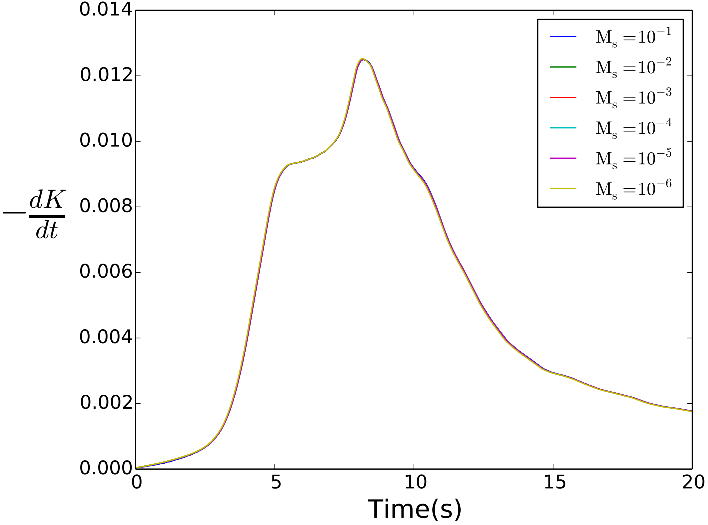
$$u_y = -\cos x \sin y \cos z$$

$$u_z = 0$$

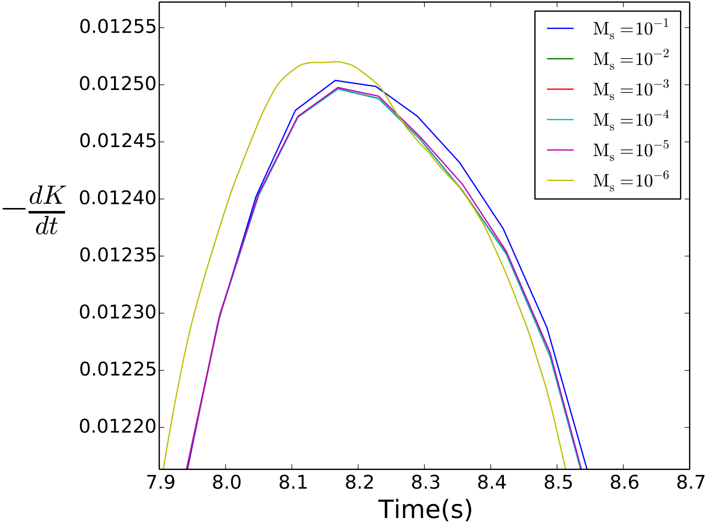
- Vary $\mathrm{M_s}$ by varying $\mathrm{P_0}$
- Evolve vortex until $t = 20$, measuring kinetic energy loss.



Taylor Green Vortex Results



Taylor Green Vortex Results



The End.

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(...but see I. Baraffe's talk for stellar applications!)