RAY TRACING ALGORITHMS

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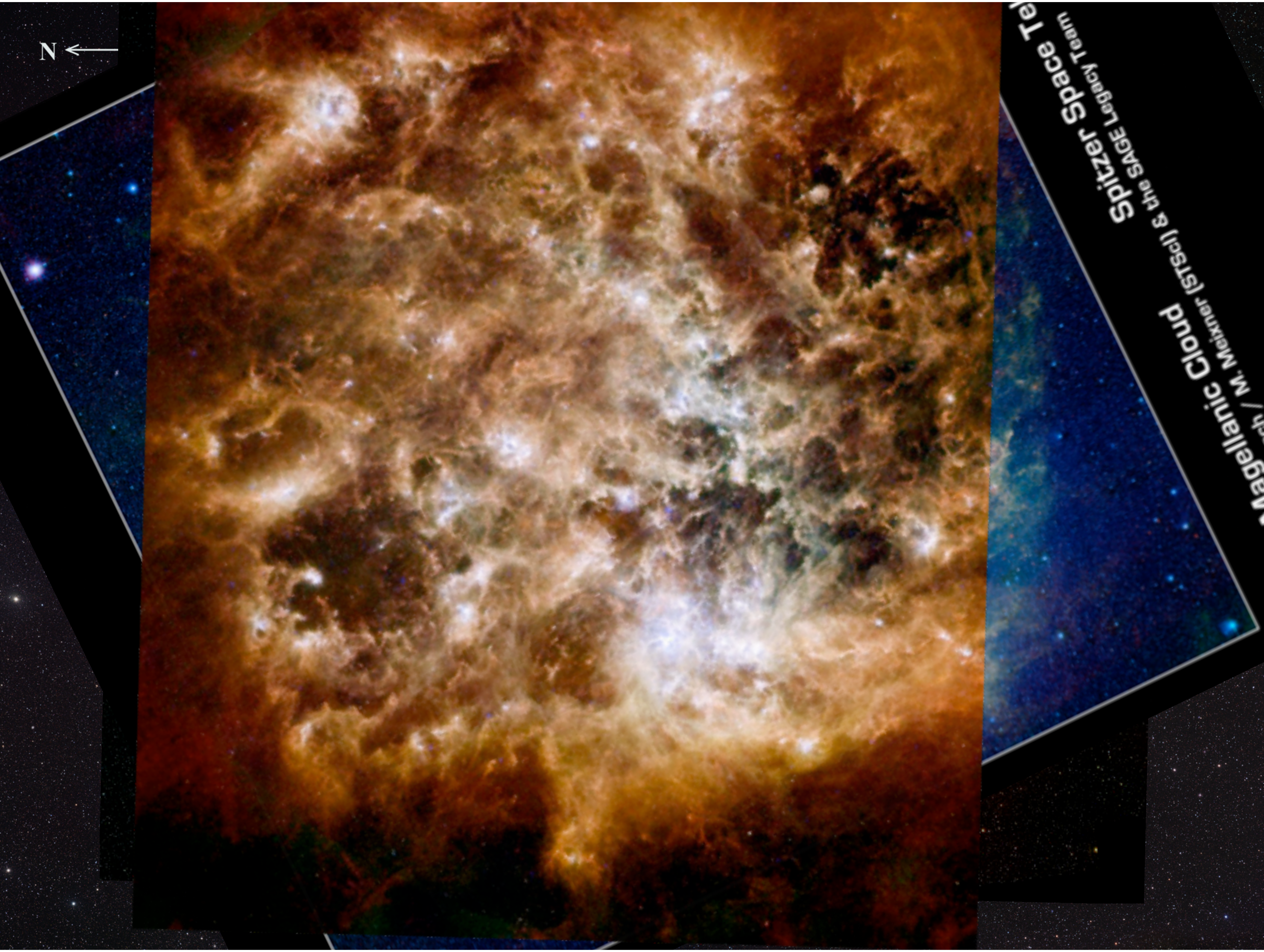
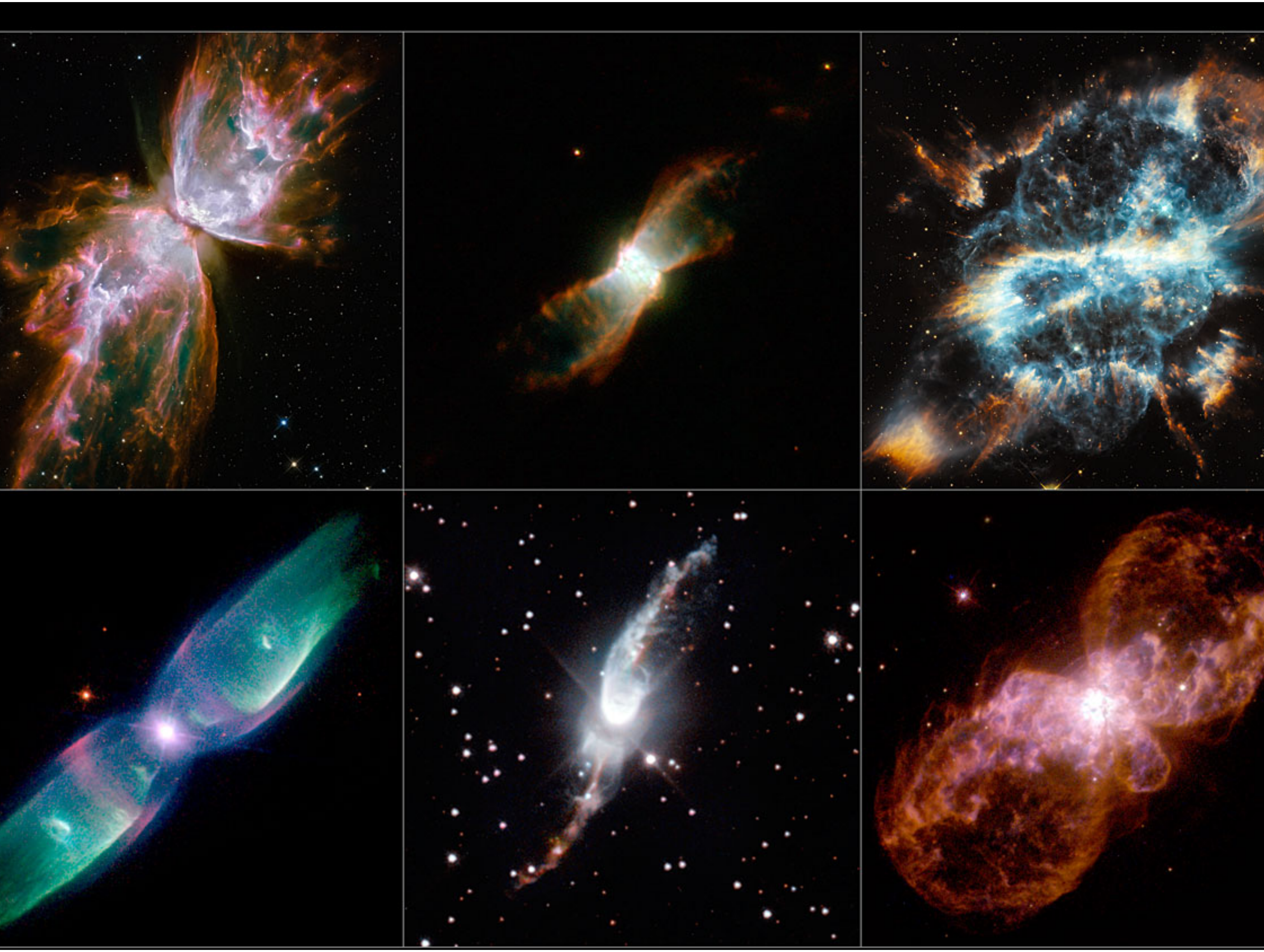
OUTLINE

- Motivations
- Boltzmann Equation
- Long Characteristics
- Adaptive Ray Tracing
- Beating the linear scaling with sources
- Computational Geometry & Beam Tracing
- The End


CALIFORNIA NEBULA, NGC1499
500 pc = 1,500 light years away
30 pc long
Xi Persei, منكب mankib, Shoulder of Pleiades:
O7.5III
330,000 solar luminosities
~40 solar masses, Teff=3.7e4K


N
Spitzer Space Tel
Magellanic Cloud
M. Meixner (STScI) & the SAGE Legacy Team



BOLTZMANN EQUATION

- All the systems we encounter in astrophysics and cosmology usually are best described through time dependent Boltzmann equation.
- f = f(**x**, t) gives the density of particles per phase space volume dx^3dv^3
- I.e. model fluid in phase space. Liouville's theorem: incompressible fluid (if there are no formation or destruction terms on the right)

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_x f + \dot{\vec{v}} \cdot \nabla_v f = \text{interactions}$$

- 6+1 dimensions
- Radiation: straight lines with speed c. Same phase space expressed with 3 spatial coordinates, two angles and energy.

BOLTZMANN TRANSPORT

- Often used in supernovae (and bomb) modeling
- Discretizes phase space, typically into angles, and energies on three dimensional spatial grid.
- Same number of operations independent of solution, including the cases where radiation intensity is zero in most directions and locations.
- Completely inadequate for highly anisotropic fields such as generated by compact sources

LONG CHARACTERISTICS FOR POINT SOURCES

- Trace ray from source to every cell on grid?

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \frac{\partial I_\nu}{\partial r} = -\kappa I_\nu$$

- Monte Carlo approach (Ciardi, Wood, Sunrise, and many more)
  - Send random photon packages from source in random directions and absorb them by a probability according to optical depth.
  - Scattering handled easily by scattering photon packages based on scattering optical depth
  - Quite easy to program a first version.
  - However, enormous number or refinements and "tricks" to help reduce inherent sqrt(N) noise.

ATTENUATION EQUATION

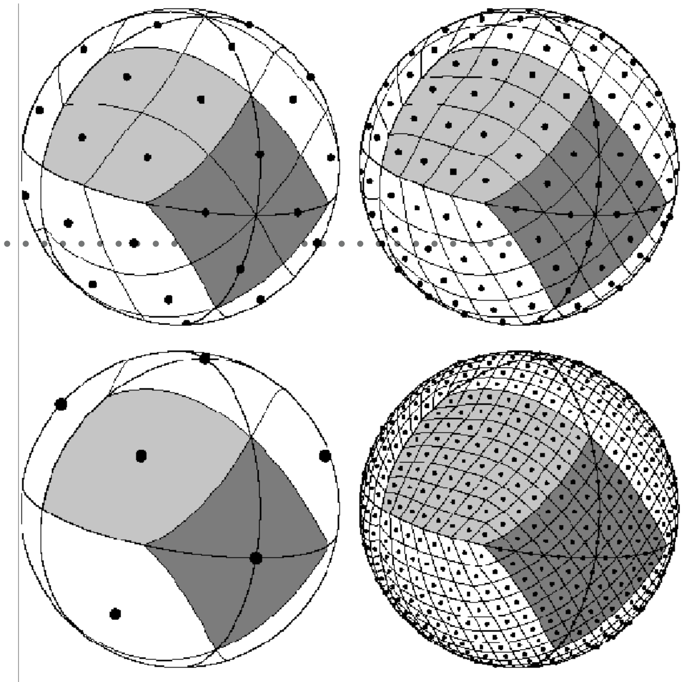
- Early work assumed light fronts travel much faster than rest of the physics (fluid flow) and only solved the attenuation equation neglecting the time dependence in the transfer equation.

$$\frac{\partial I_\nu}{\partial r} = -\kappa_\nu I_\nu \qquad \frac{1}{c}\frac{\partial I_\nu}{\partial t} + \frac{\partial I_\nu}{\partial r} = -\kappa I_\nu$$

- Piecewise constant optical depth just leads to exponential decay in Intensity
  - $I_1 = I_0 \exp(-tau)$

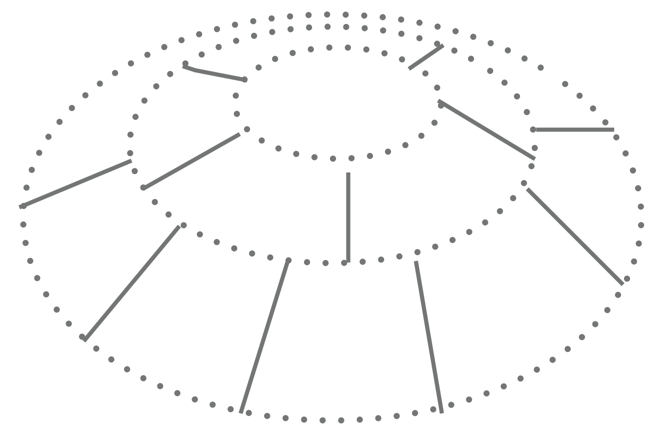
PIXELIZATION OF THE SPHERE

- Choose rays so that the associated area is approximately equal. I.e. each ray has similar weight.
- Healpix
  - Equal area, hierarchical, nice indexing, pix_2vec and inverse
  - Enables adaptivity
- Current work using triangulations with straight edges rather than curved. Still hierarchical but more suited to computational geometry



Healpix

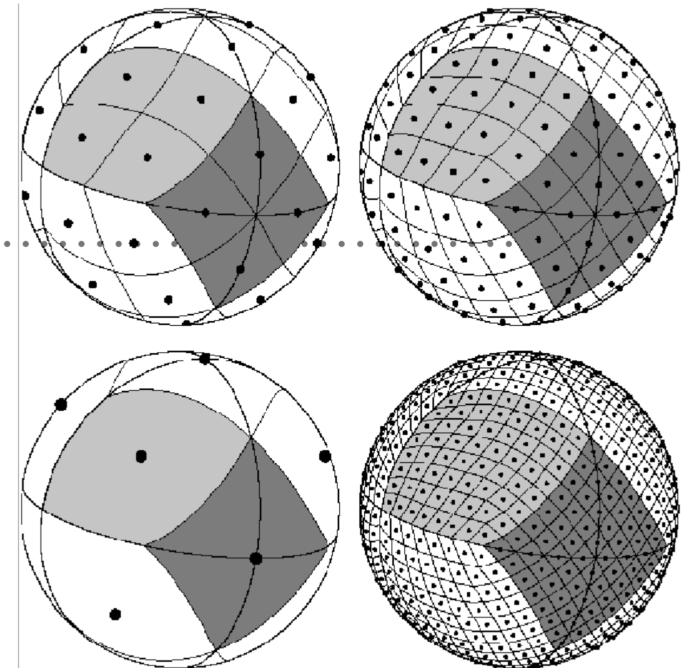
Abel & Wandelt 2002



Igloo

Abel, Norman & Madau 99

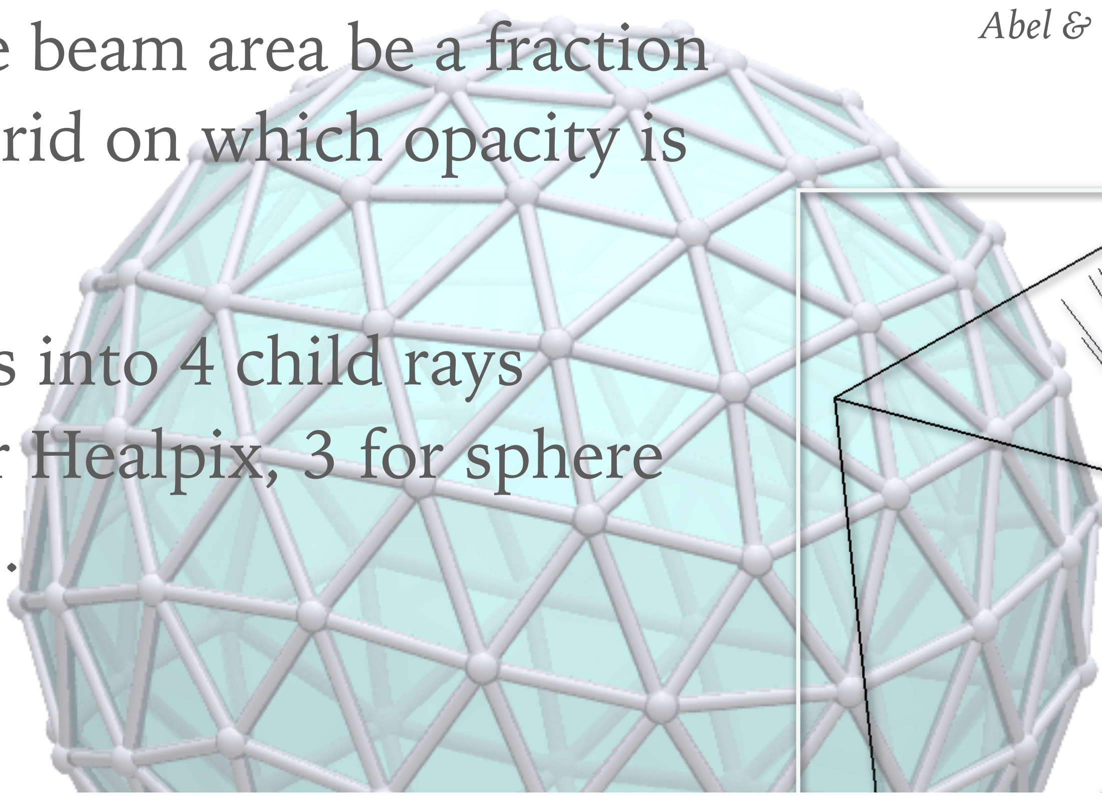
ADAPTIVE RAY TRACING

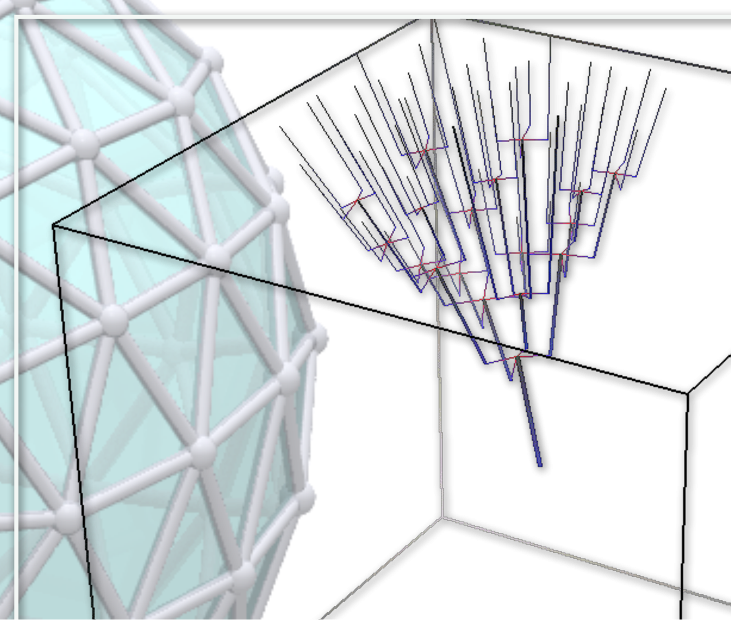


Healpix

Abel & Wandelt 2002

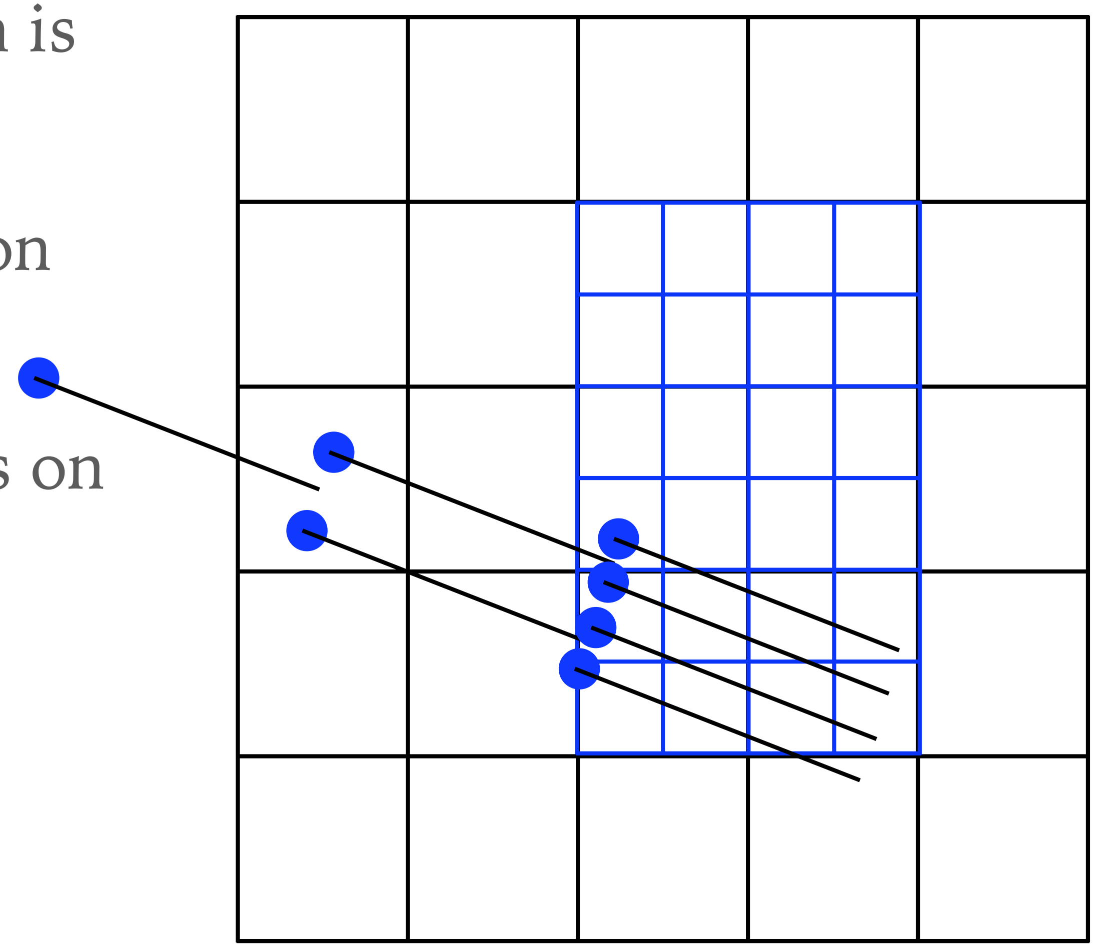
- Start at low levels of the hierarchy. I.e. say with 12 rays at the source. Beam area associated with ray grows with square of radius.
- Typical refinement strategy (splitting criterion) have beam area be a fraction of dx^2 of the grid on which opacity is given.
- Each ray splits into 4 child rays recursively for Healpix, 3 for sphere triangulations.





ADAPTIVE RAY TRACING

- Dramatic efficiency gain as the "angular grid" created by the rays adapts to the radiation field.
- Rays can stop once radiation is absorbed.
- Not expensive when radiation doesn't travel very far
- Computational cost depends on form of solution

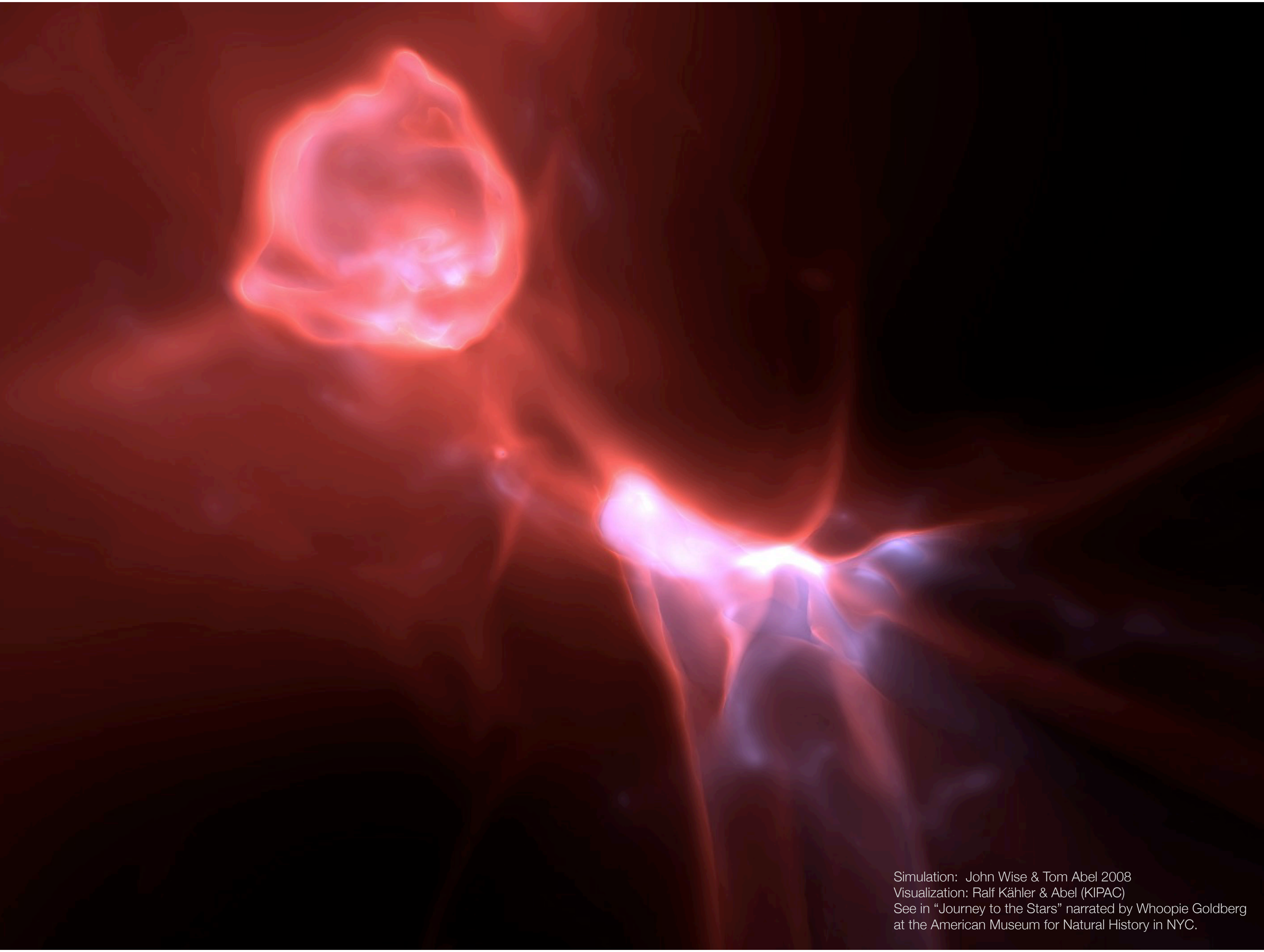


KEEPING THE TIME DEPENDENCE

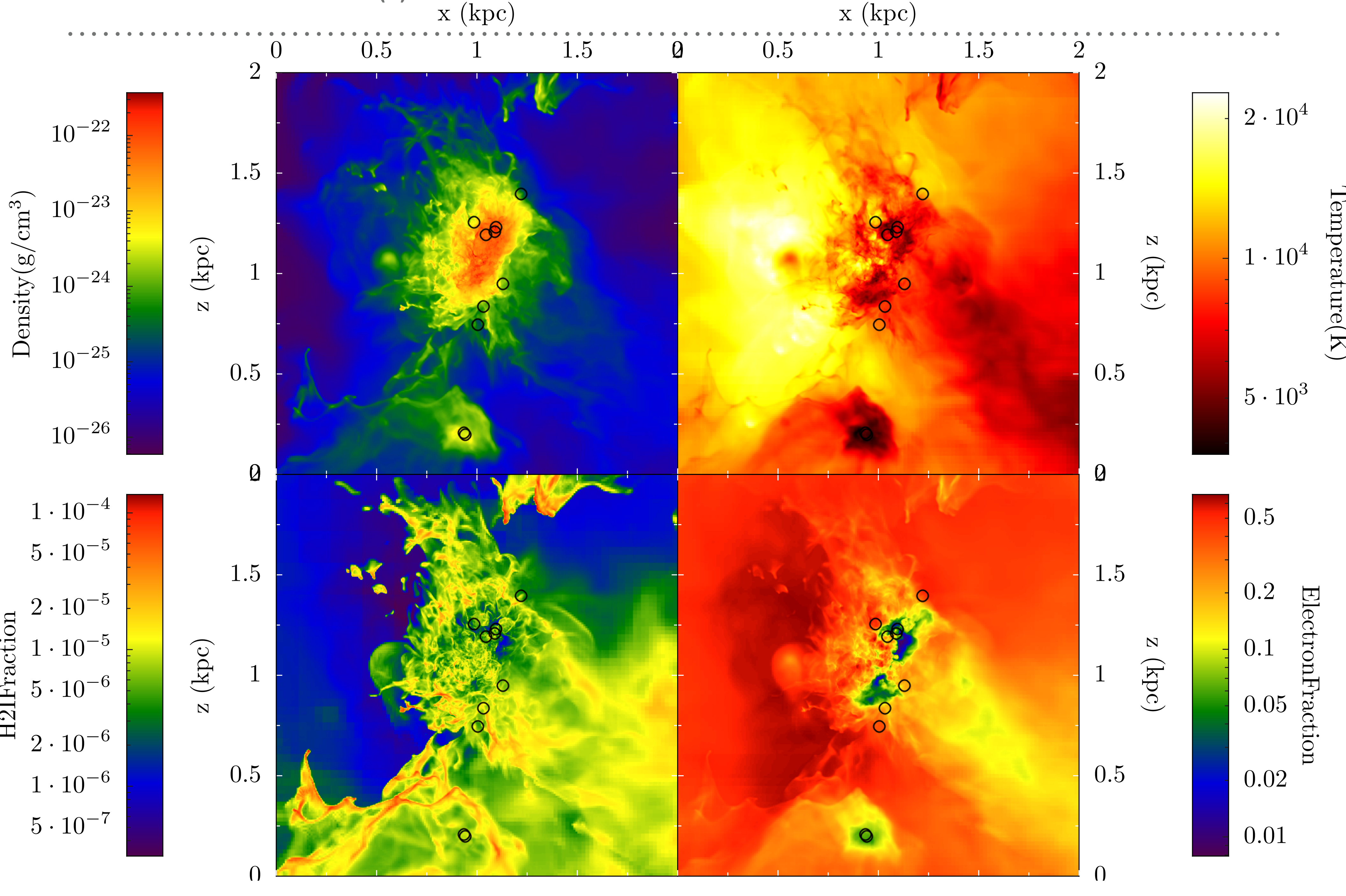
- Trace "photon packages" from source in discrete and short time intervals

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \frac{\partial I_\nu}{\partial r} = -\kappa I_\nu$$

- Photons travel as far as time step allows with their specified speed.
- Photon packages carry as many photons as the source emits into the beam of the ray in one photon time step.
- Subcycle chemistry+radiation within hydro-timestep

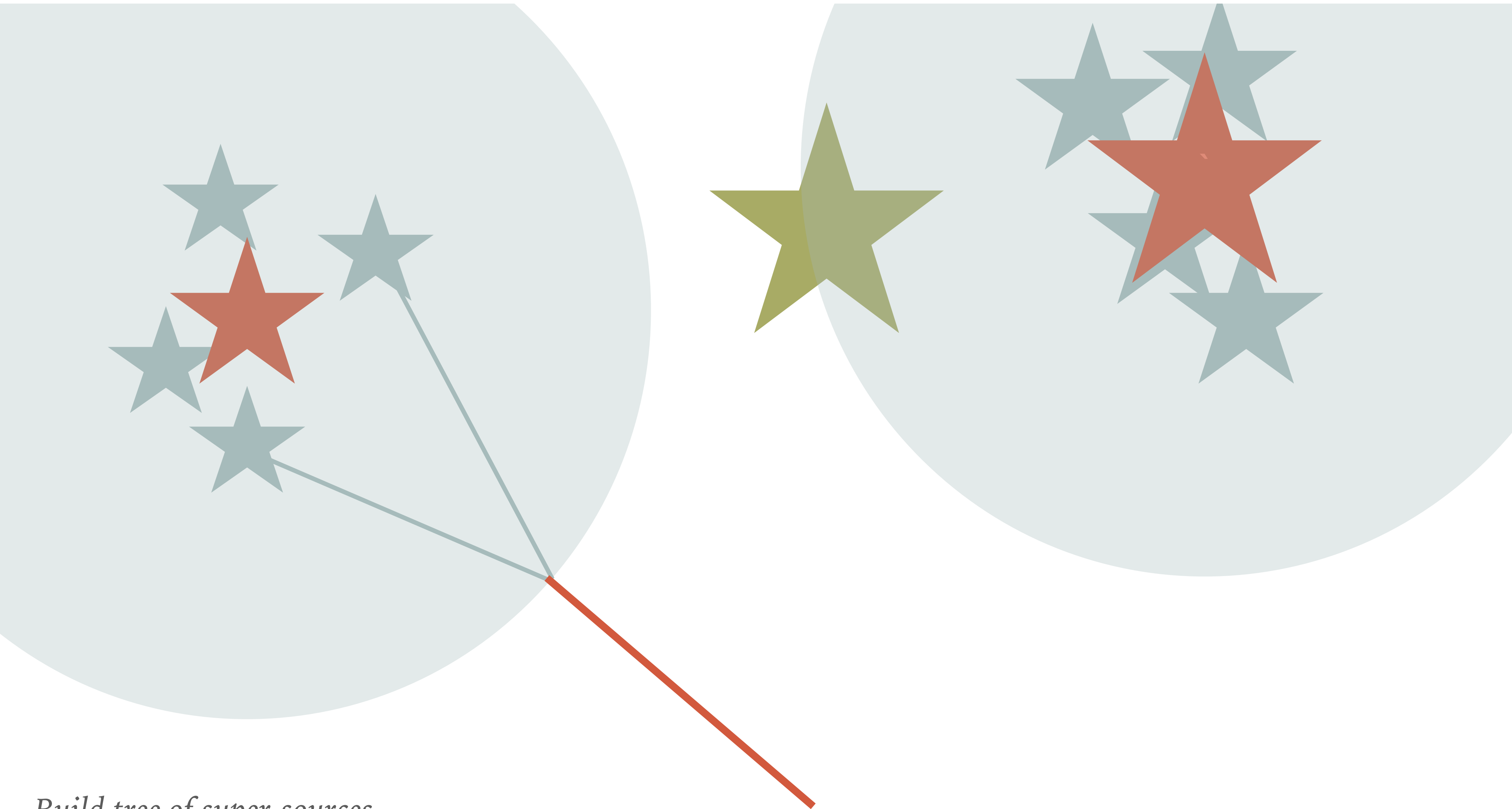

Simulation: John Wise & Tom Abel 2008
Visualization: Ralf Kähler & Abel (KIPAC)
See in “Journey to the Stars” narrated by Whoopie Goldberg
at the American Museum for Natural History in NYC.

Z=8.2 IN DM HALO: 2×10^8 SOLAR MASS: 3 SOLAR MASSES ACCRETED TOTAL (!) ON 25 SEED BLACK HOLES



COST SCALES LINEARLY WITH THE NUMBER OF SOURCES!?

- Adaptive ray tracing is done for every radiation source
- Expensive for many sources.
- However, we can and have beat this down to log(N) sources via ray merging.



Build tree of super-sources

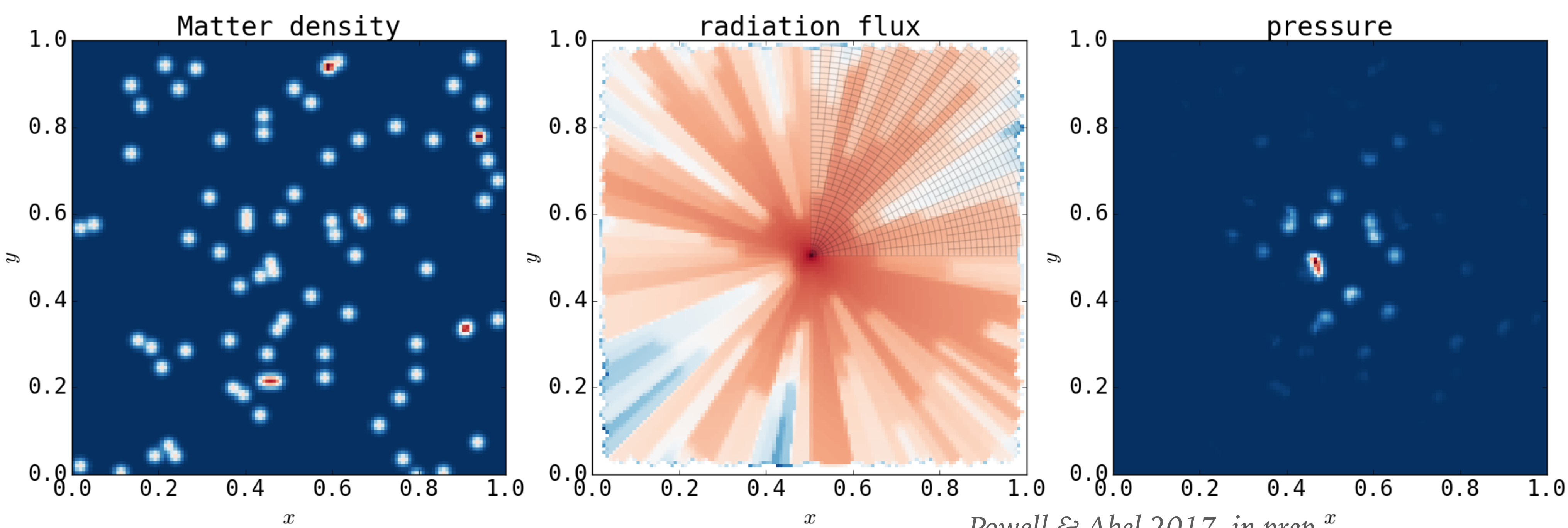
Rays halt at a fix radius from their associated super source (one level up in tree)

Rays check whether they are in the same pixel on the sphere of the SS

If they are they can combine and the added new ray intensity now travels radially from SS

BEAM TRACING

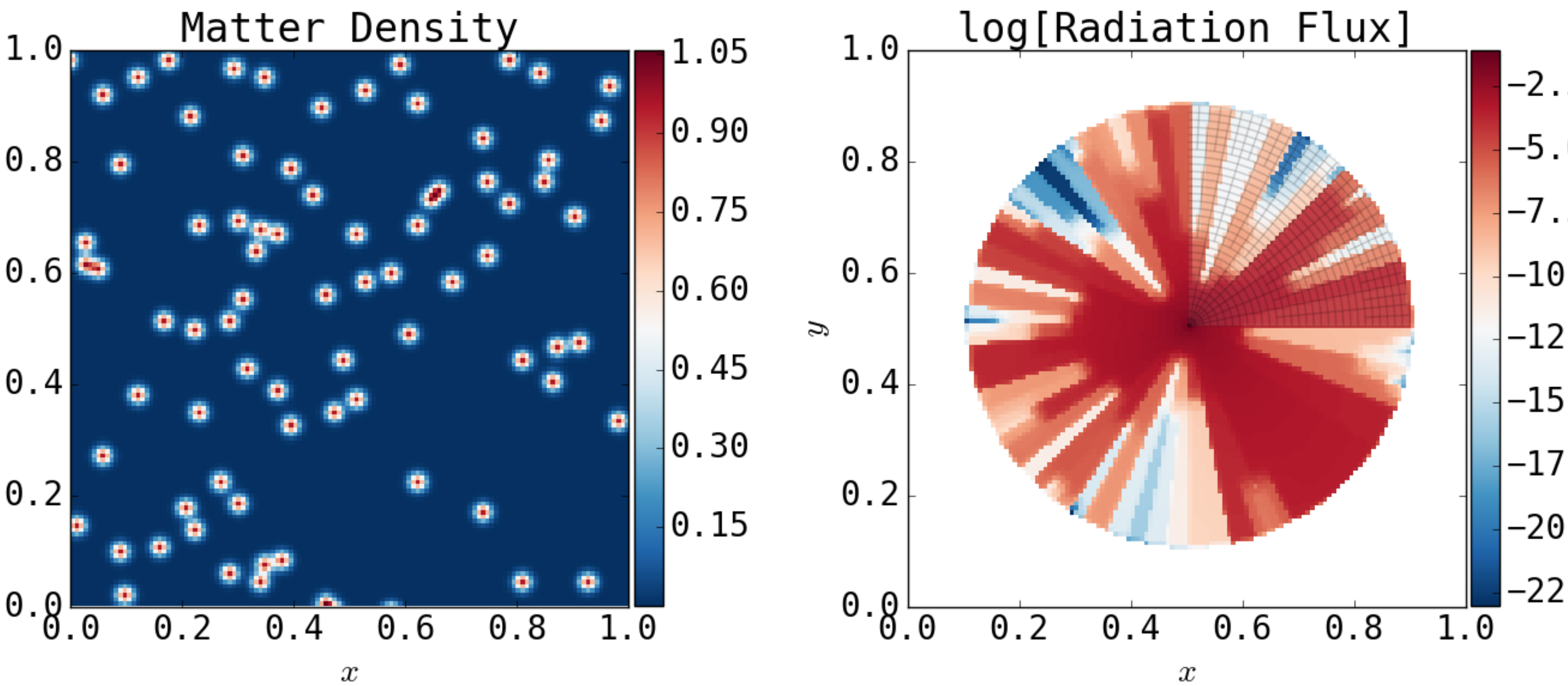
- Instead of tracing single one dimensional trace entire beam through volume.
    - Improve accuracy for same number of rays
    - Have much fewer rays for same accuracy. -> Less communication and more floating point ops.



Powell & Abel 2017, in prep.[x]

BEAM TRACING

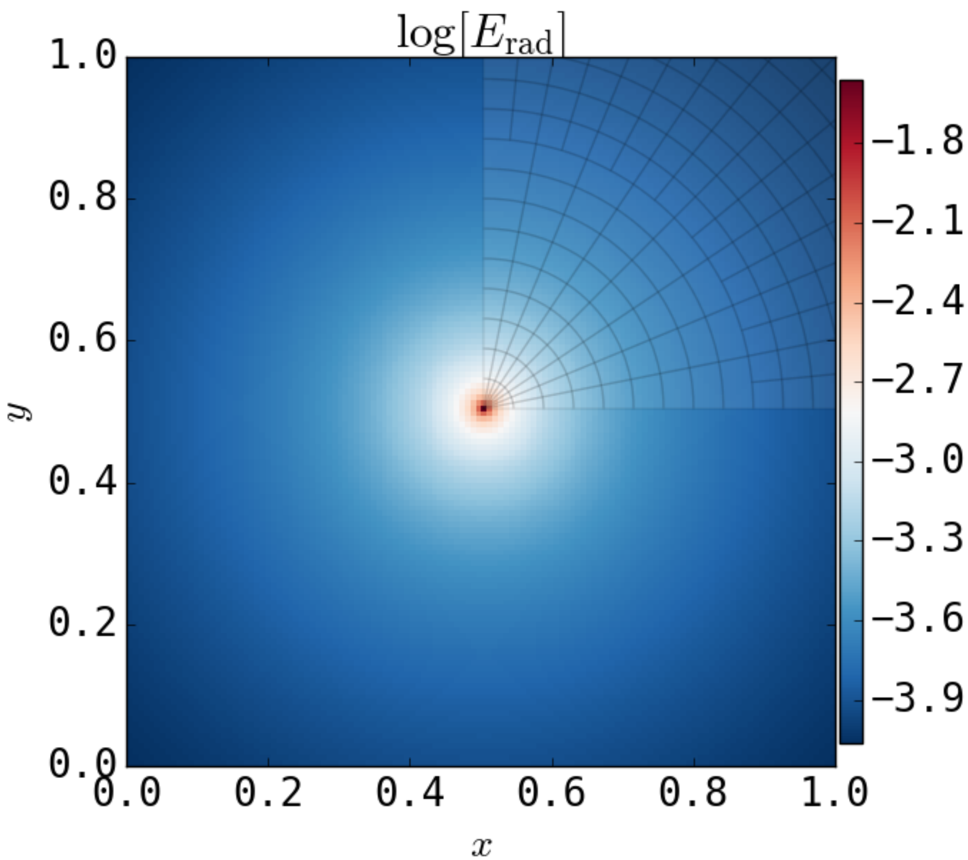
- Coarse. Beam < 4 dx

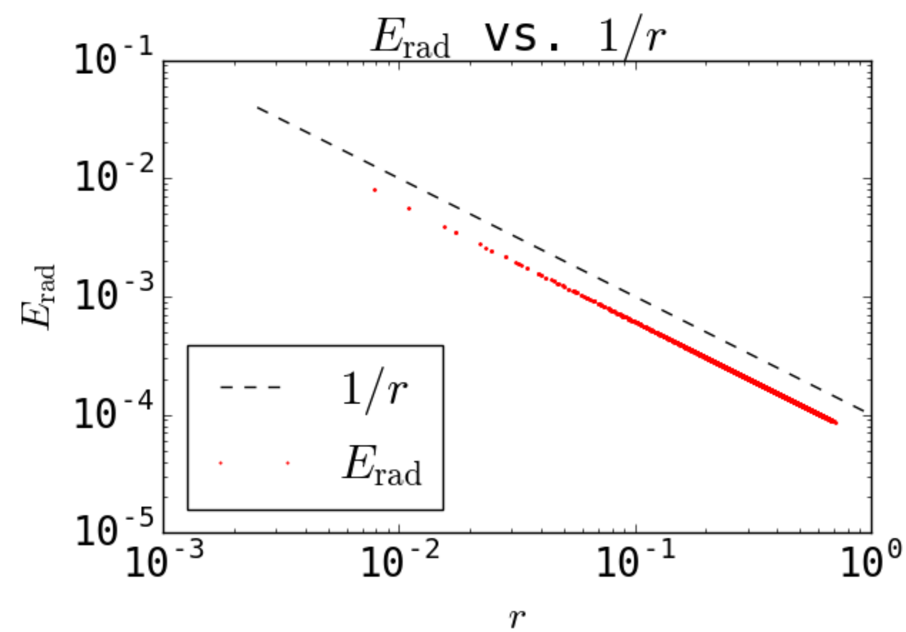


Powell & Abel 2017, in prep.

EXACT RADIAL DEPENDENCE

- Perfect symmetry

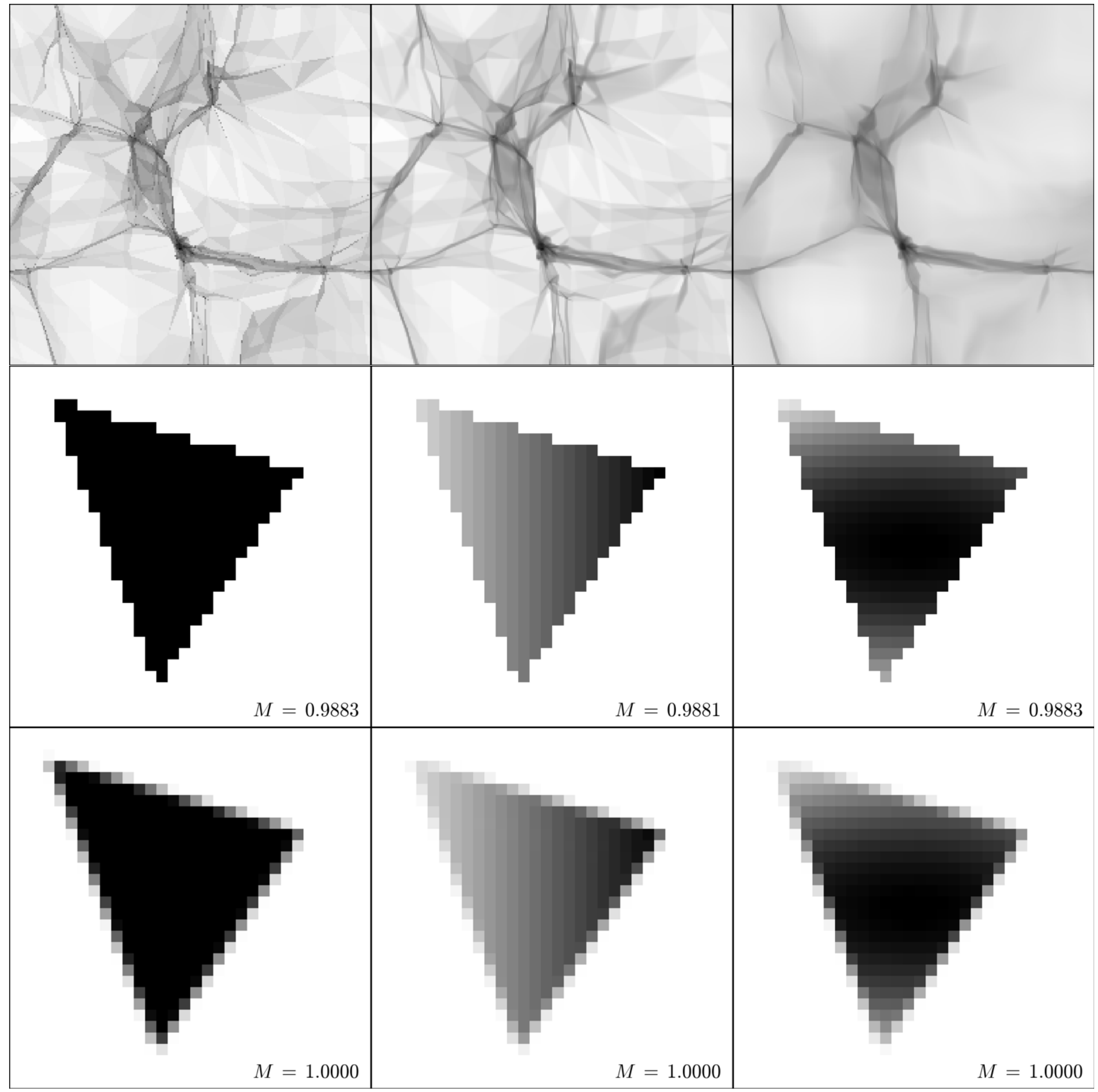




Powell & Abel 2017, in prep.

EXACT OVERLAP INTEGRALS. USEFUL BLACK BOX?

- To write new cosmology and plasma codes exploiting cold three dimensional manifolds in 6D phase space one needs to learn how to calculate exact overlap integrals.
- This leads to exact and robust remeshing techniques which likely much more generally useful.
- Started to use these ideas to do beam tracing for radiation transport.
- Enables waterbags in higher dimensions.
- New hydro methods?



Powell & Abel 2016, JCompPhys

SUMMARY

- Quick overview over some typical ray tracing approaches
  - Abel, Norman and Madau (1999), Ciardi et al (2001-), Abel & Wandelt (2002), Mellema et al (2006), Wise and Abel (2008-), Pawlick & Schaye (2008, 2011), etc.
- Knowing how to calculate exact overlap integrals for arbitrary polyhedra likely to be useful in many applications, including for beam tracing.
- Because of the short lifetime of massive stars, significant UV output in galaxies comes from relatively few sources. We'll see many nice examples today I believe.