Multidimensional upwind methods on unstructured grids

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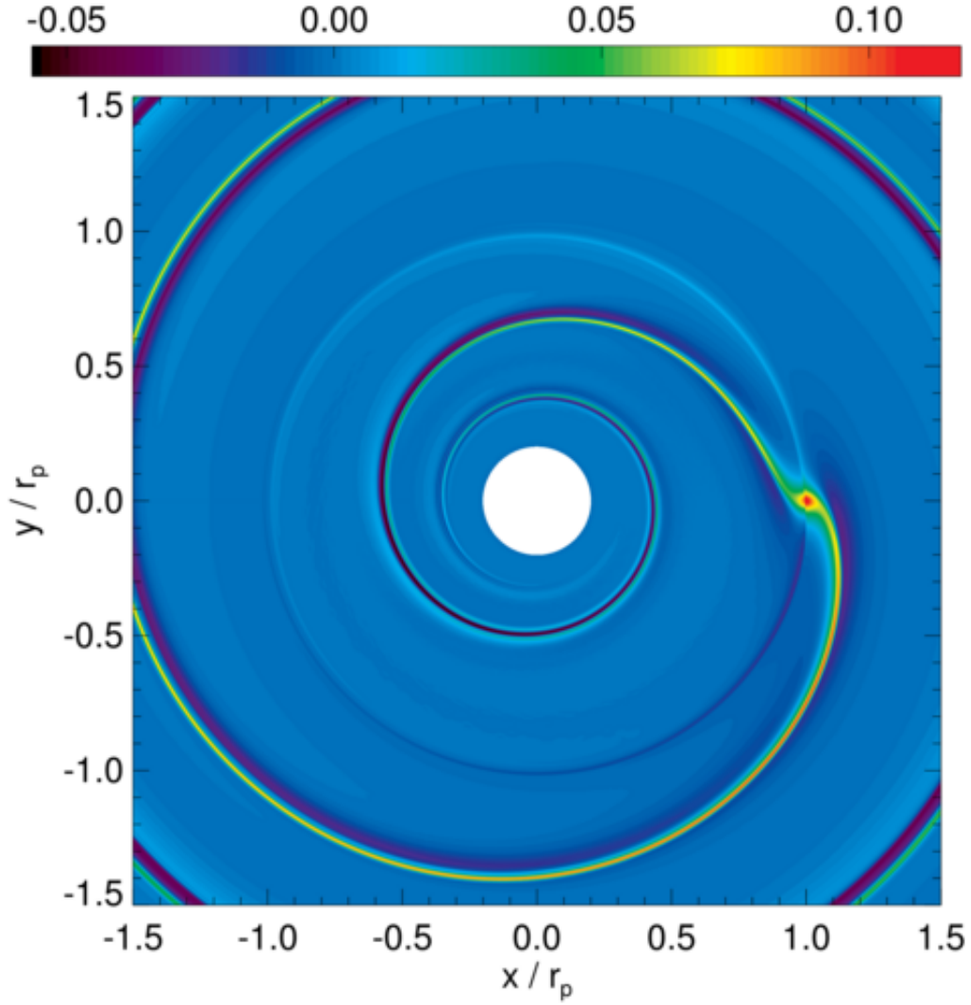
THE ROYAL SOCIETY

ASTRIX

What interests me…

- Planet formation
- Disc-planet interactions
- Disc hydrodynamics



Baruteau et al. 2014

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My usual tool…

- Roe solver, 2nd order (flux limiter)

  Roe (1981), Eulderink & Mellema (1995), Paardekooper & Mellema (2006)

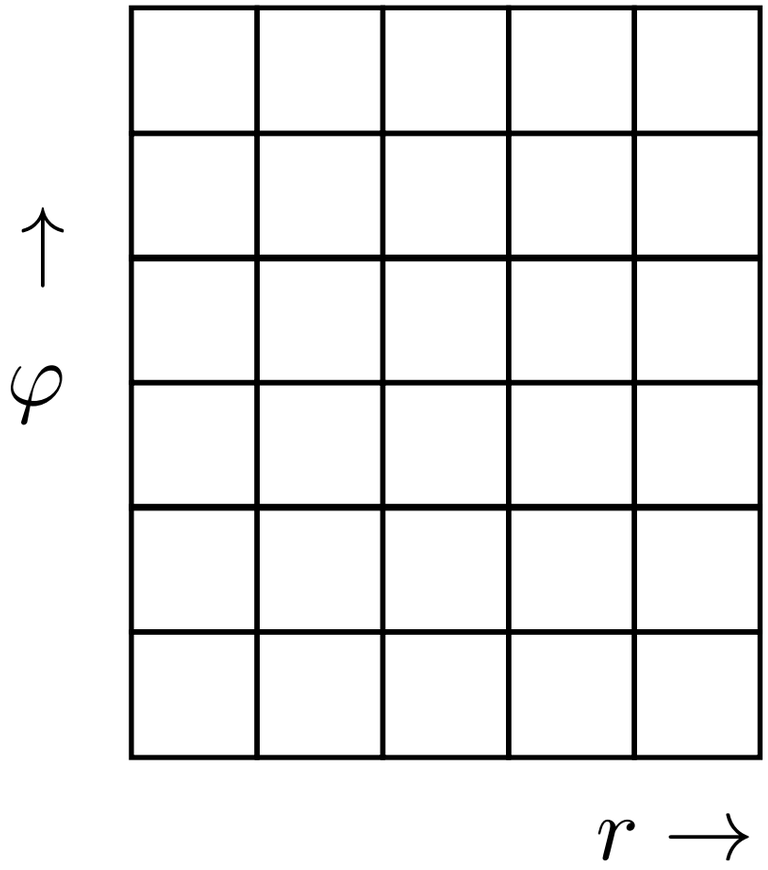
- Dimensionally split / unsplit

  Leveque (2001)

- Source terms: stationary extrapolation

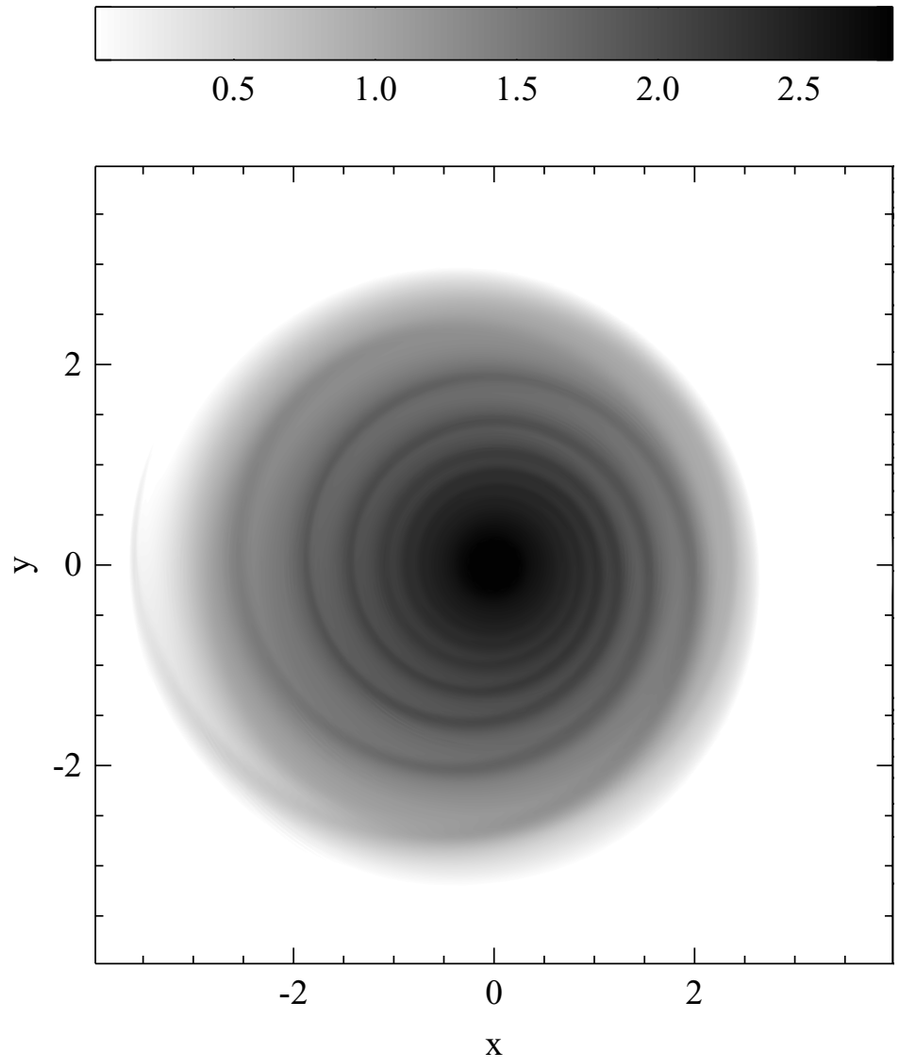
  Eulderink & Mellema (1995)

- Rectangular mesh in cylindrical coordinates



What frustrated me I

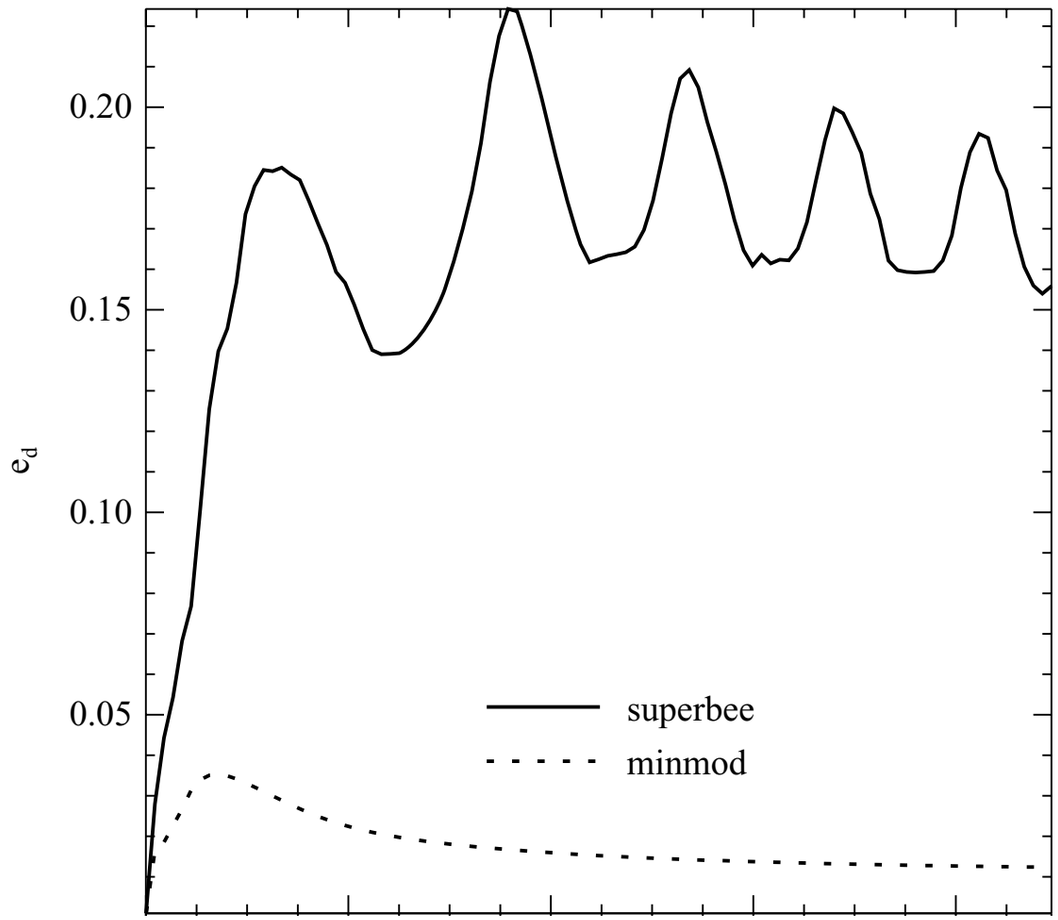
- Planet formation in close binaries
- Disc reaction to companion
- Gas disc eccentricity critical for planetesimal evolution



Paardekooper et al. (2008)

What frustrated me I

- How eccentric does the disc get?
- Depends on flux limiter…
- Superbee more in line with e.g. FARGO, but minmod is converged

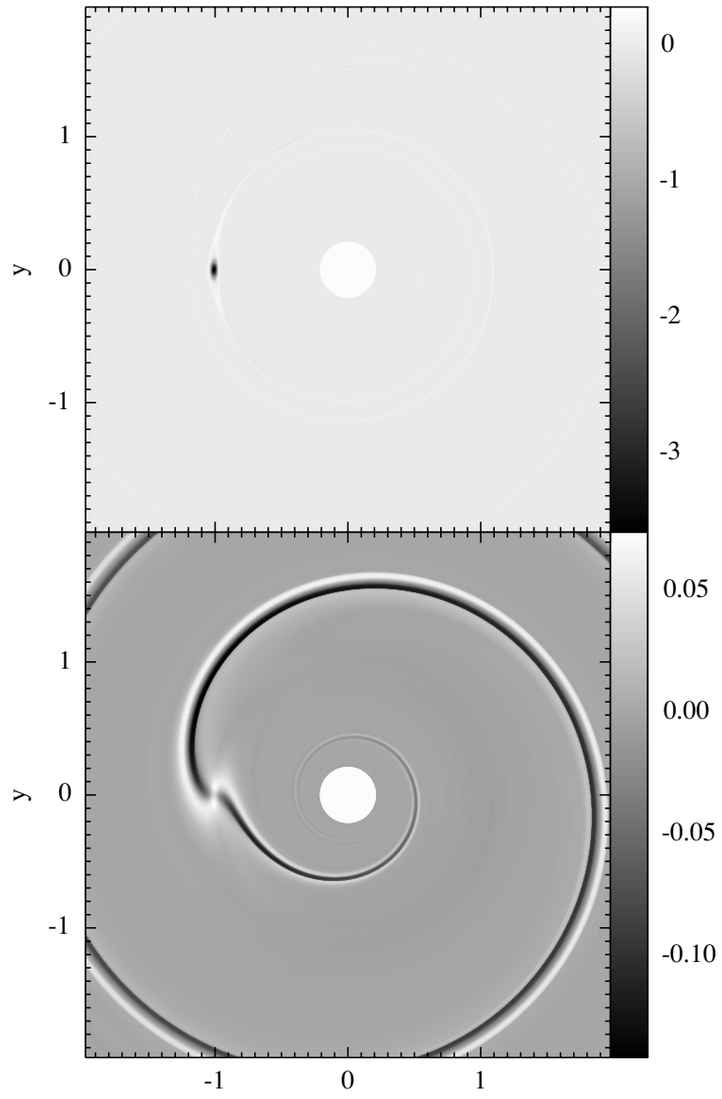


Paardekooper et al. (2008)

What frustrated me II

- Vortex migration in discs
- Vortices can trap solids: building sites for planets?

  e.g. Barge & Sommeria (1995)
- Vortices emit density waves
- Angular momentum transport leads to migration



[x] Paardekooper et al. (2010)

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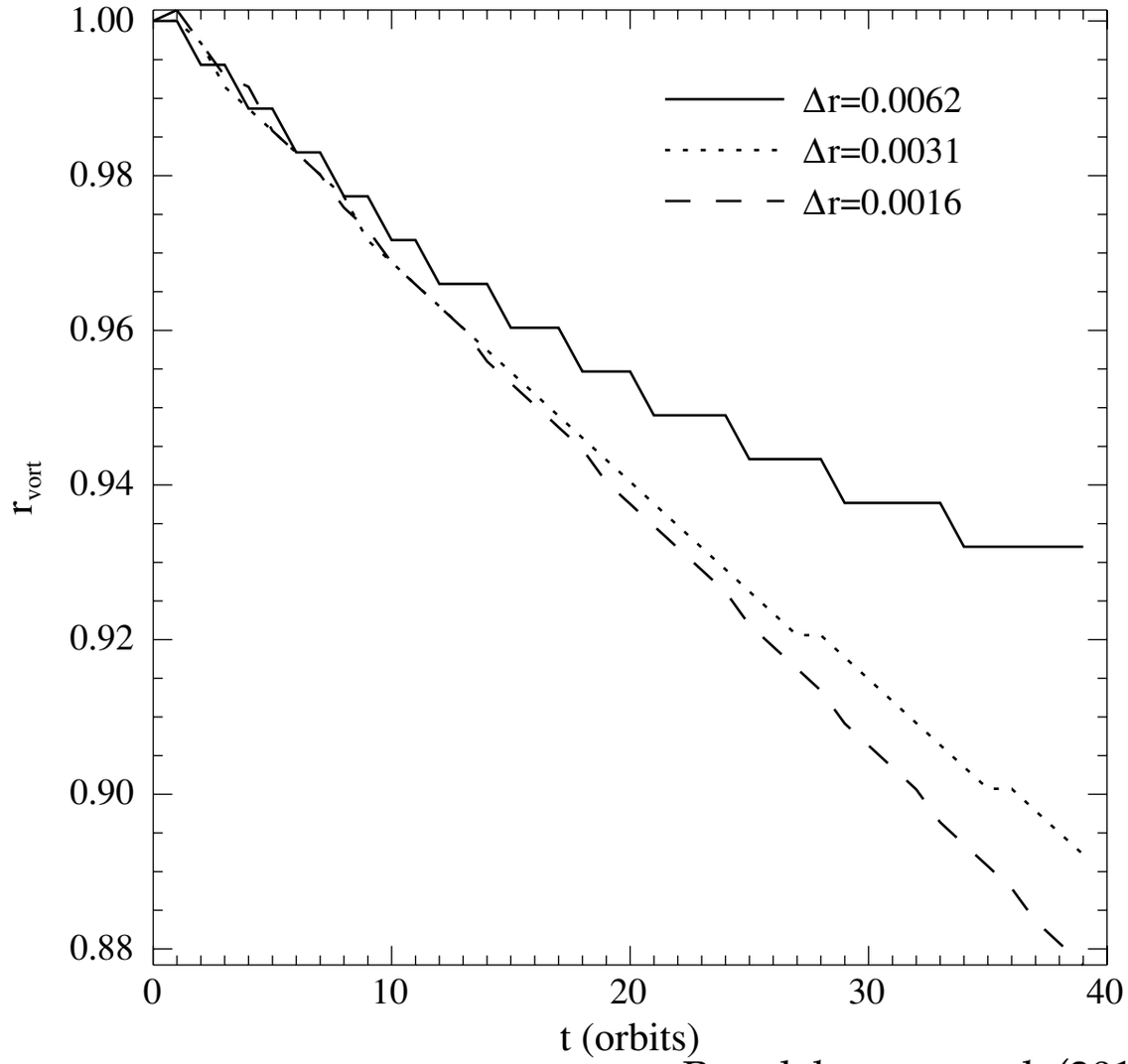
What frustrated me II

- How fast does a vortex migrate?

- Vortex dissipates through numerical diffusion

- Need crazy resolution



Paardekooper et al. (2010)

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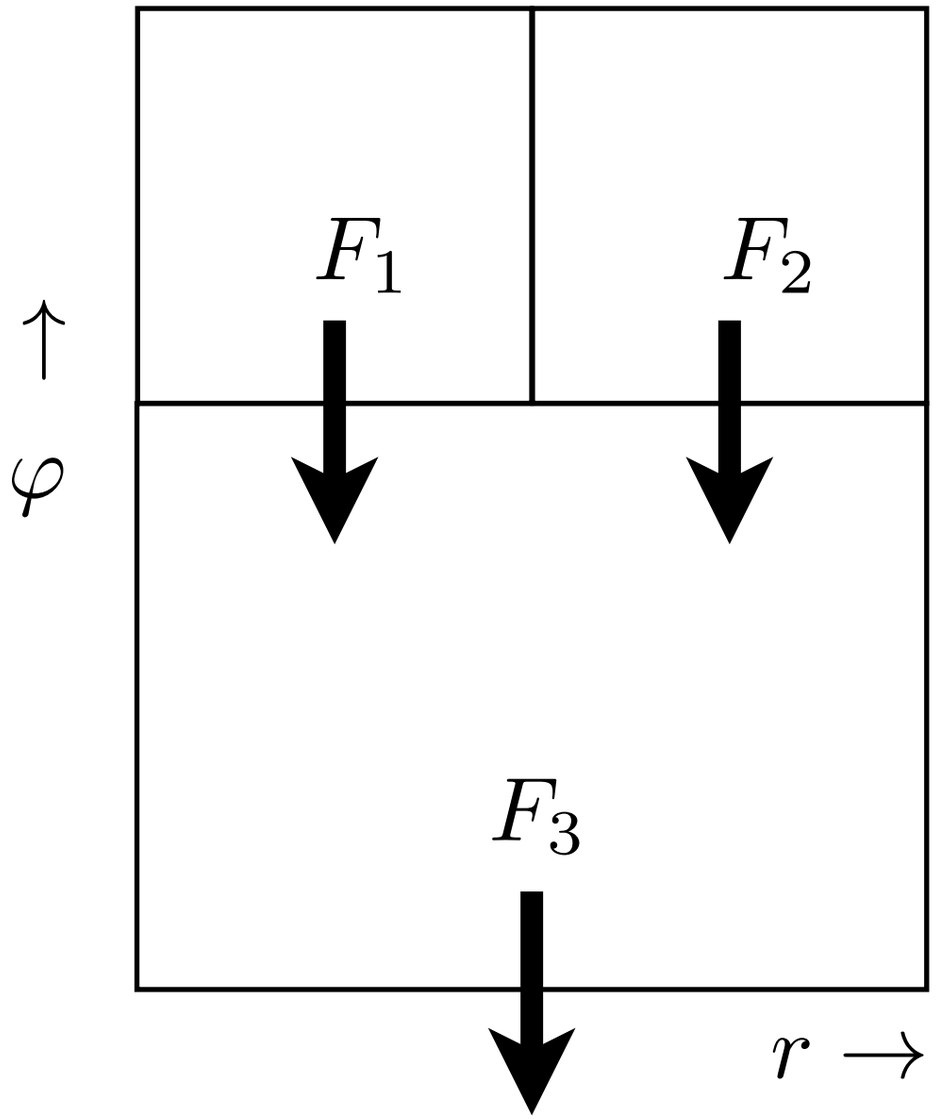
What frustrated me III

- Off-the-shelf AMR is difficult
- Unperturbed disc usually has

$$F_1 + F_2 \neq F_3$$

- Error in angular momentum flux



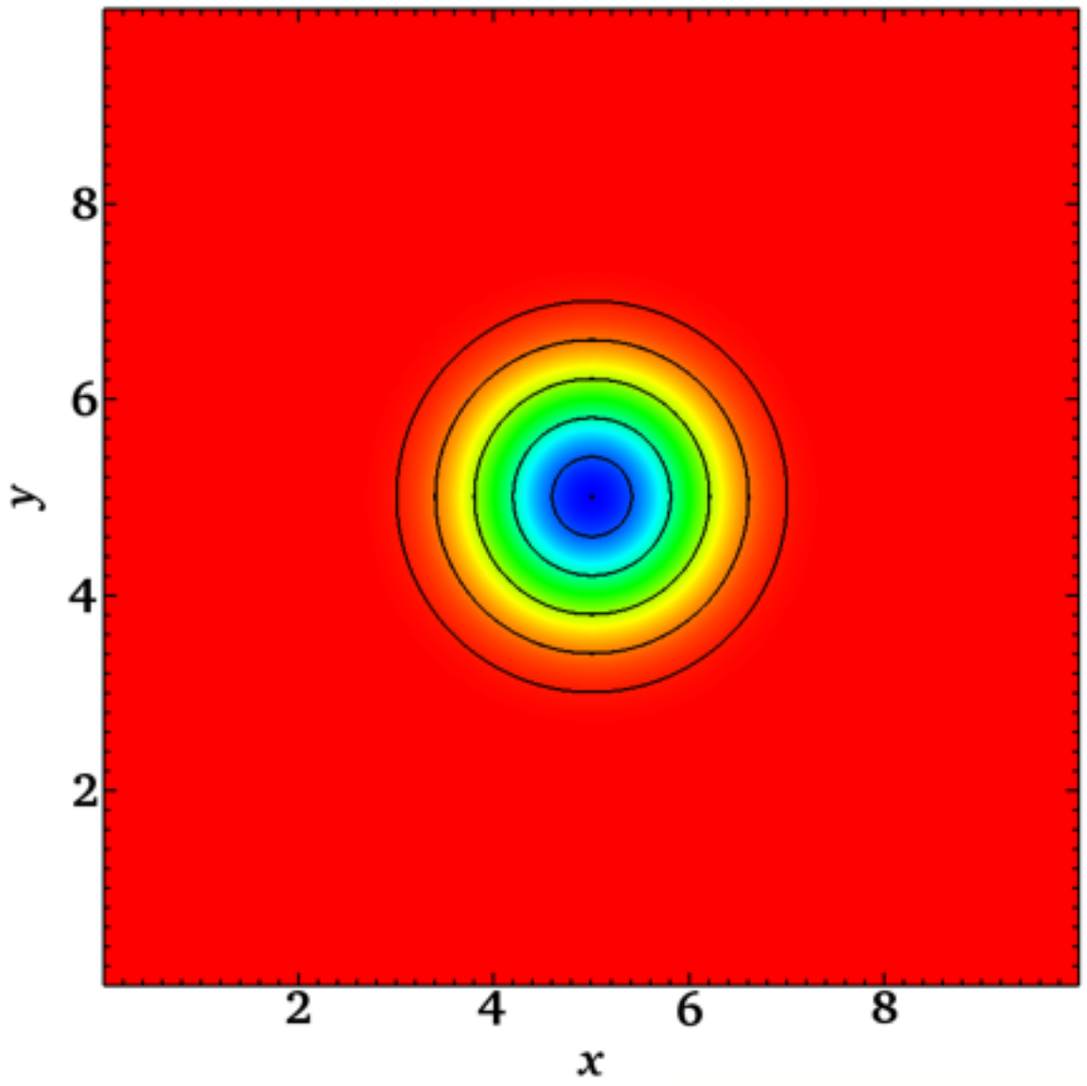
Test problem

- 2D isentropic vortex

  Yee et al. (1999)

- Stationary solution to inviscid equations

- Numerical solution for $t \to \infty$: no vortex….



Stationary solutions

- In 1D, a related issue arises when integrating sources:

$$\frac{\partial q}{\partial t} + a\frac{\partial q}{\partial x} = -2x(q - Q)$$

- Small perturbations around a stationary state

- Well-balanced schemes (stationary extrapolation)

  e.g. Eulderink & Mellema (1995), Bale et al. (2002)

Stationary 2D solutions

- What can be done in 2D?
- Quite a few options, but to stay close to my expertise:
- Enter **Multidimensional Upwind** methods

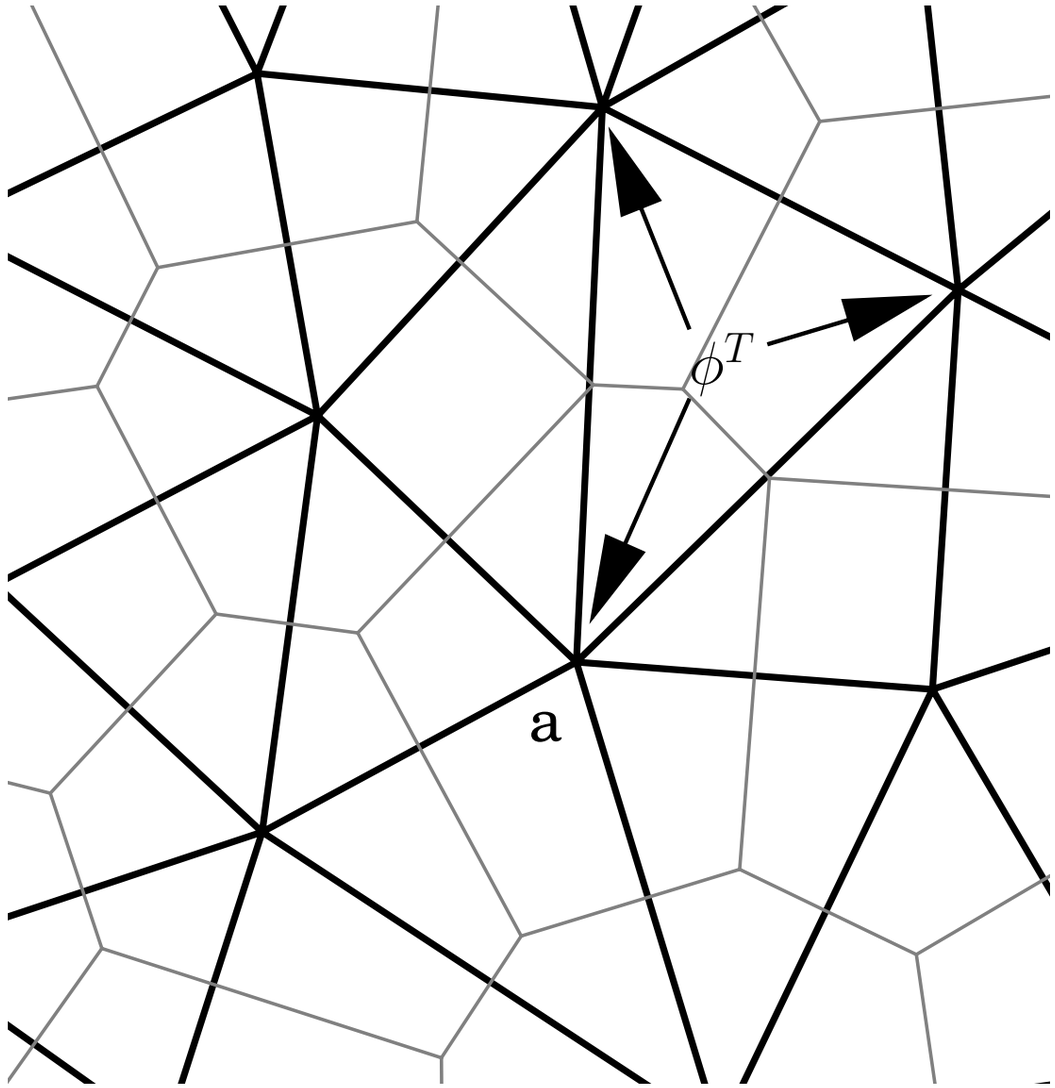
  e.g. Deconinck et al. (1993), van der Weide (1998), Abgrall (2001)

Residual distribution

- Consider a conservation law $\dfrac{\partial \mathbf{W}}{\partial t} + \nabla \cdot \mathbf{F} = 0$ on a triangulation $\mathcal{T}$
- Define the **residual** of a triangle as $\phi = \int_T \nabla \cdot \mathbf{F}$
- **Distribute** the residual over the nodes of triangle

Residual distribution

- No residual: no evolution
- Ideas developed for linear advection
- For suitable linearisation, apply to nonlinear CLs

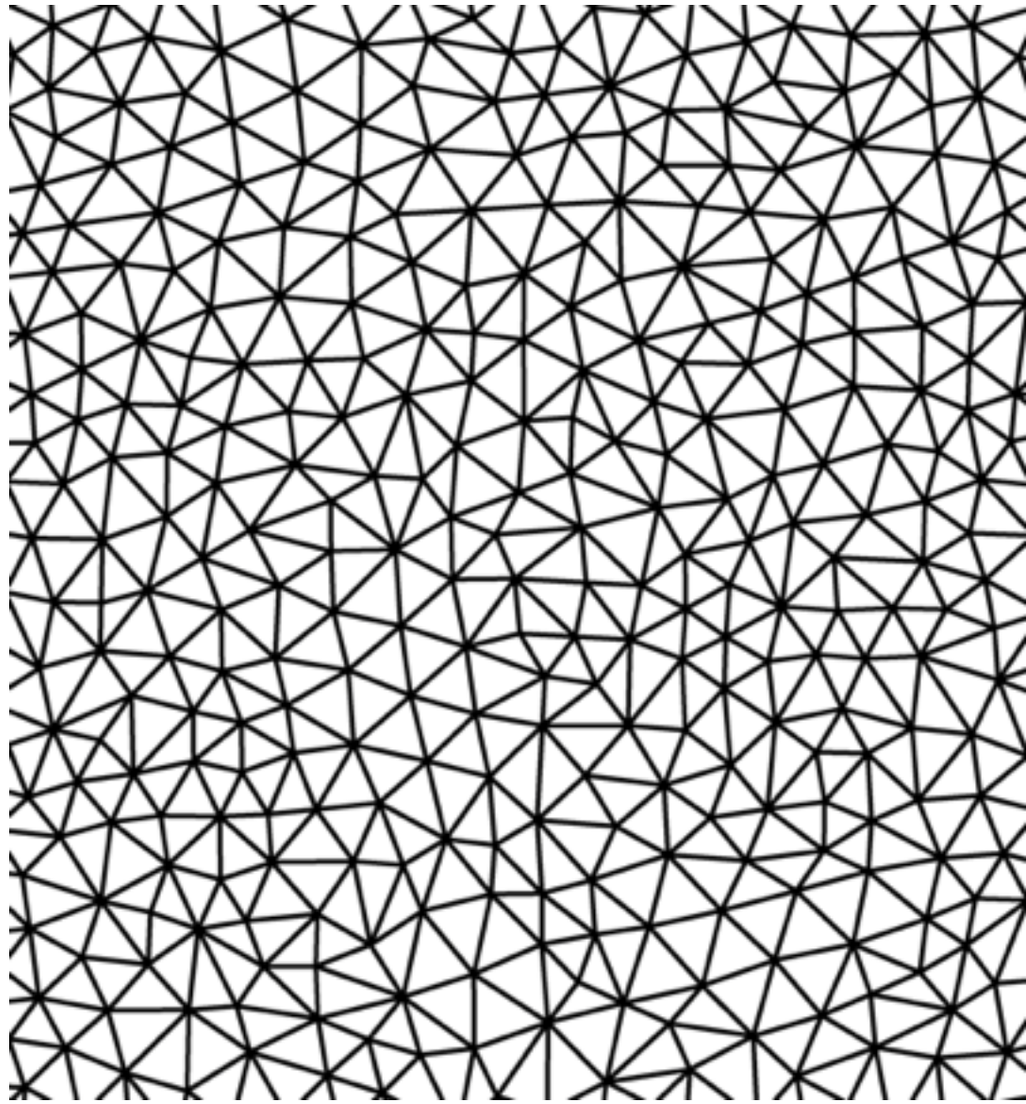


Residual distribution

- For P1 linear elements (i.e. triangles in 2D), Roe's linearisation works

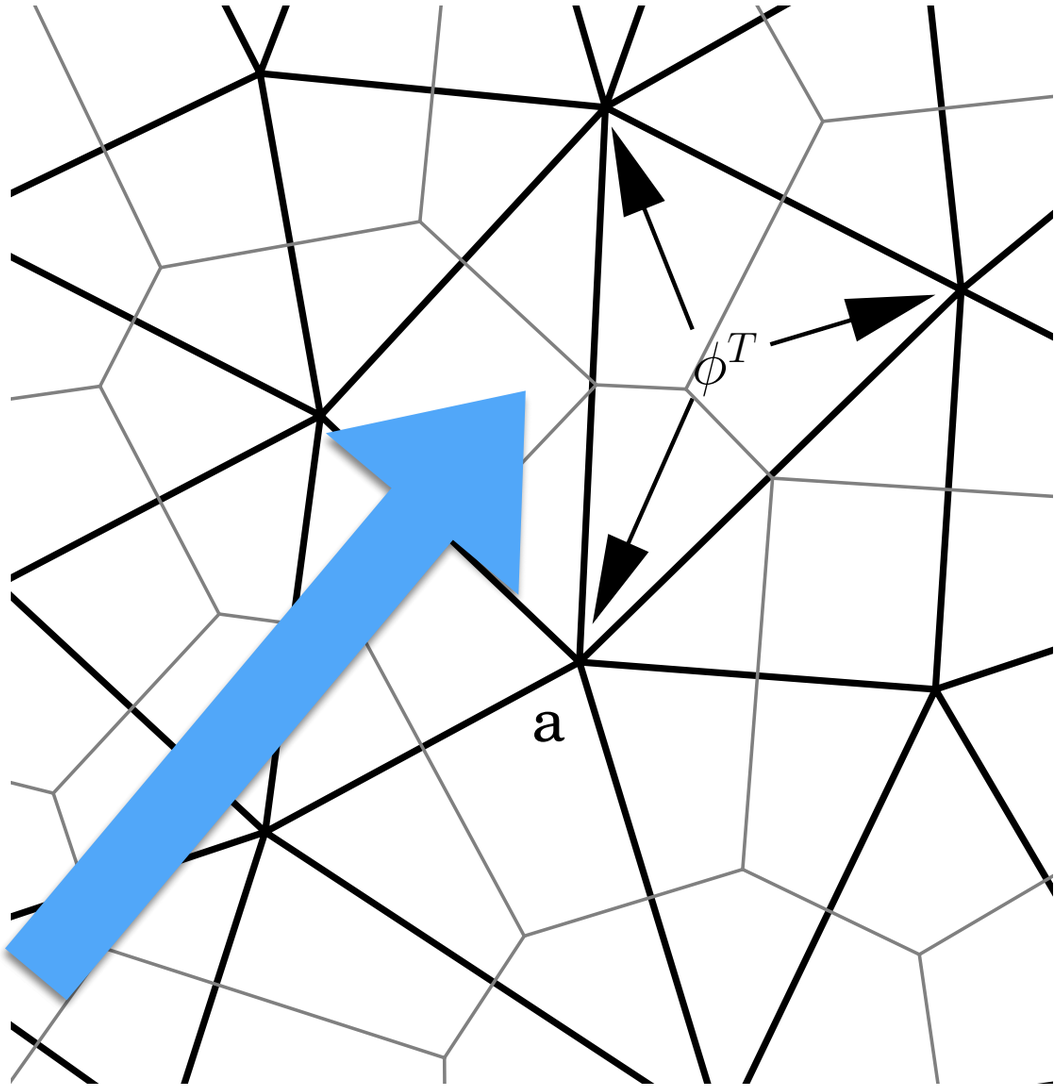
  Deconinck et al. (1993)

- Combined with multidimensional upwinding: 2D Roe solver analog



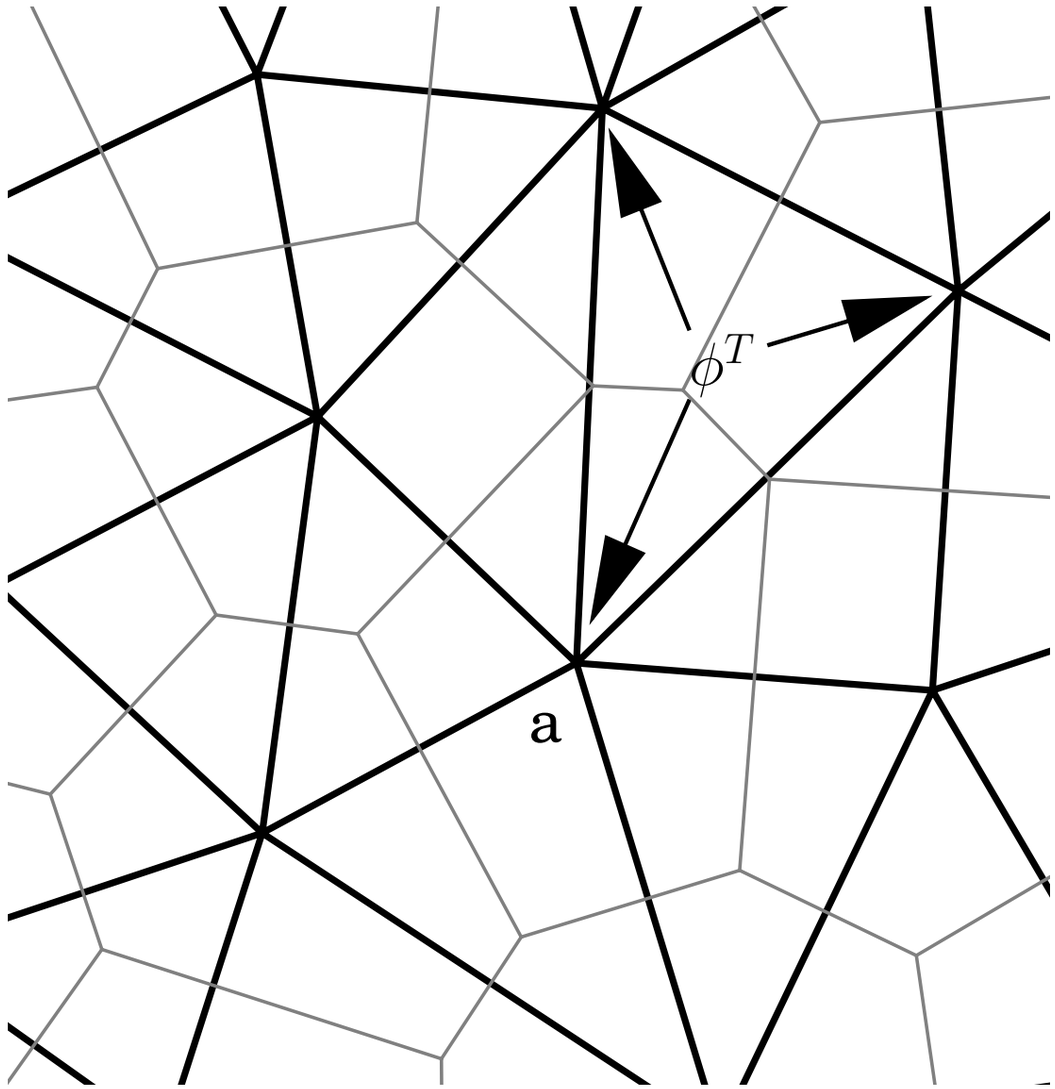
Upwinding

- How to distribute residual?
- Draw information from the proper places
- In case of linear advection: not send anything to node **a**



Distribution schemes

- Other design criteria:
  - Monotonicity (shocks)
  - Linearity preserving
- Godunov: can't do both



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- AStrophysical fluid dynamics on TRIangular eXtreme grids
- GPU implementation of explicit 2D RD for AFD

  Ricchiuto & Abgrall (2010)
- Open source on GitHub

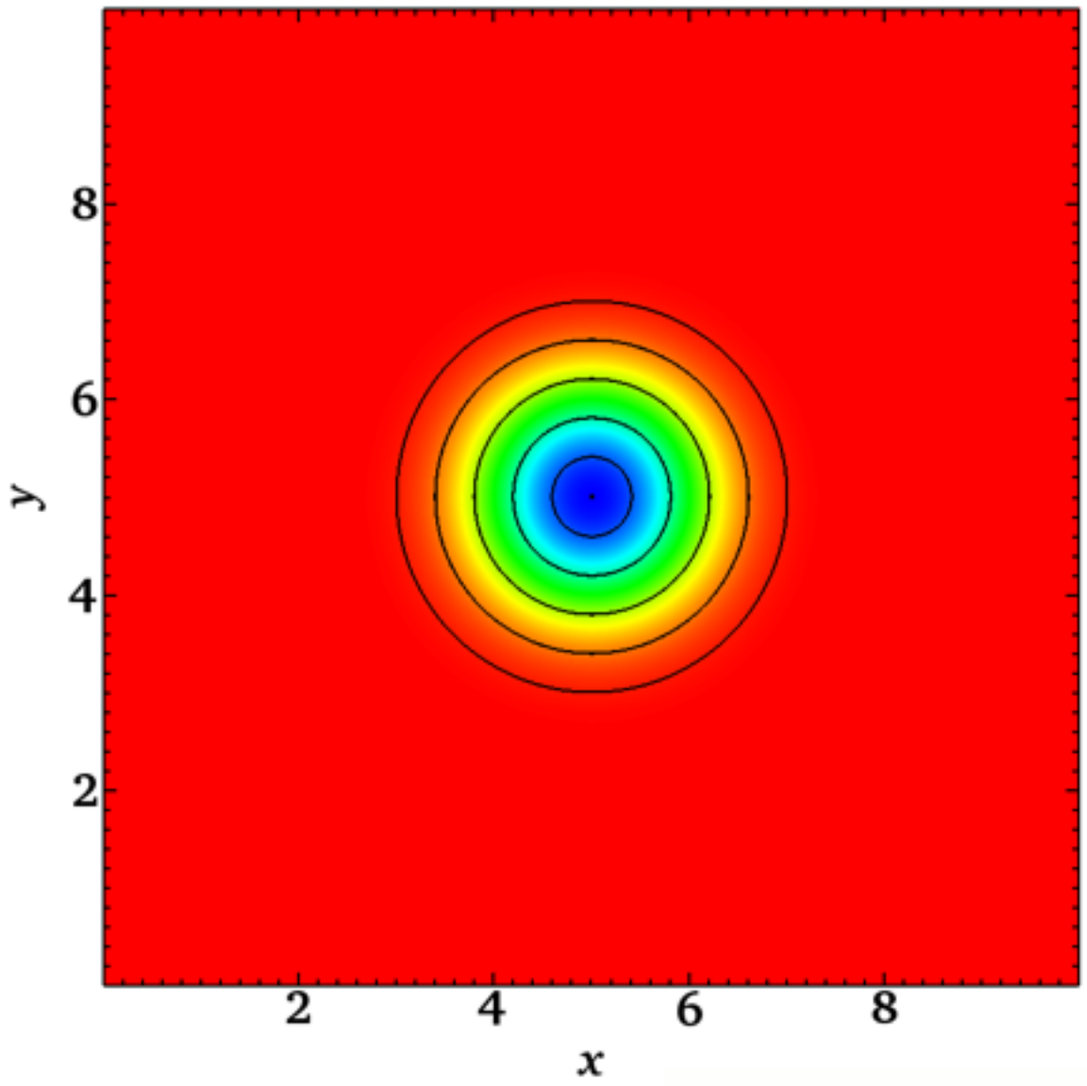
  https://github.com/SijmeJan/Astrix

Vortex problem

- 2D isentropic vortex

- Stationary solution to inviscid equations

- Numerical solution for $t \to \infty$: no vortex….



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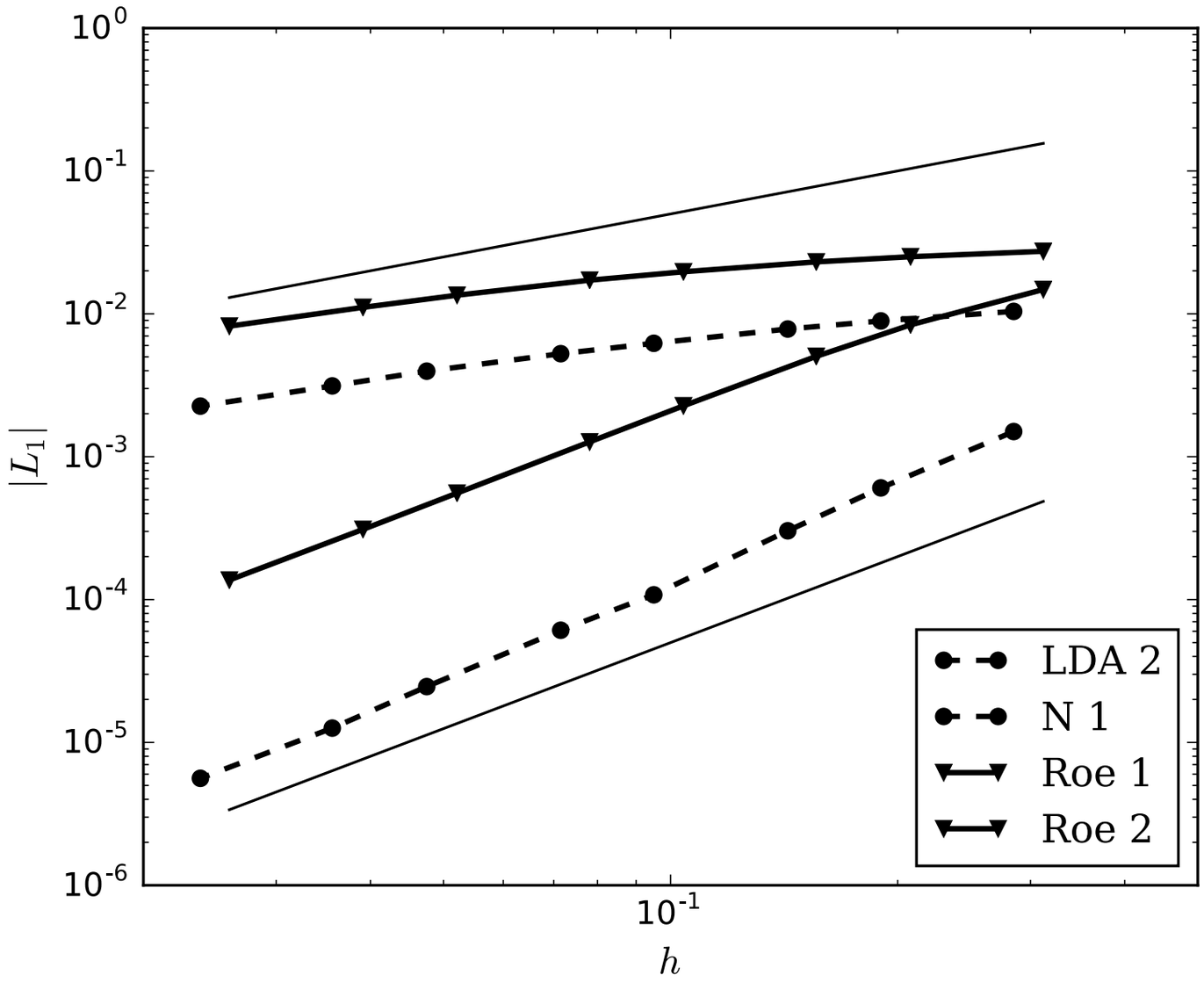
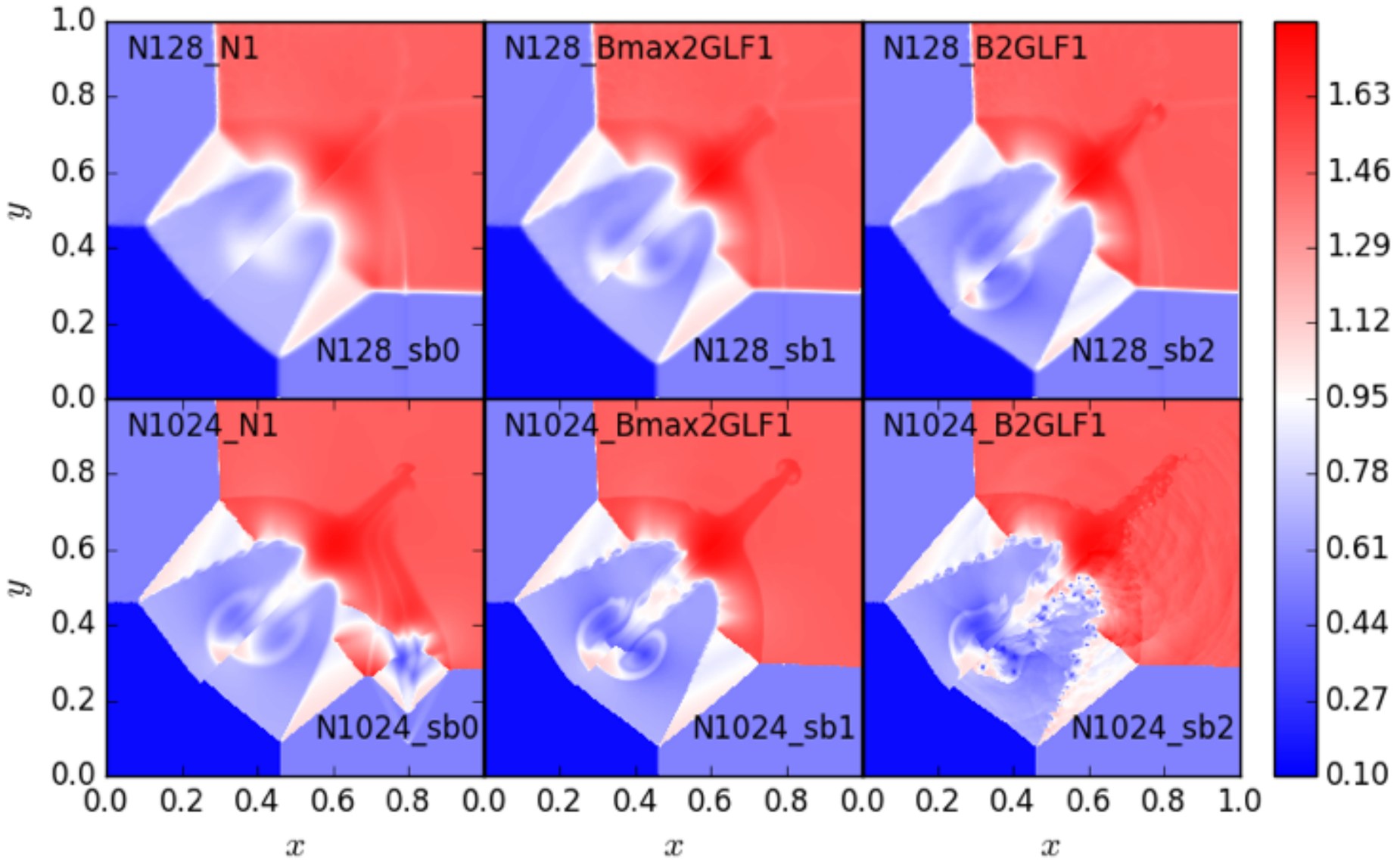
Vortex problem

- 2D isentropic vortex
- Compare Roe solver to Astrix
- L_1 density error



Riemann problem



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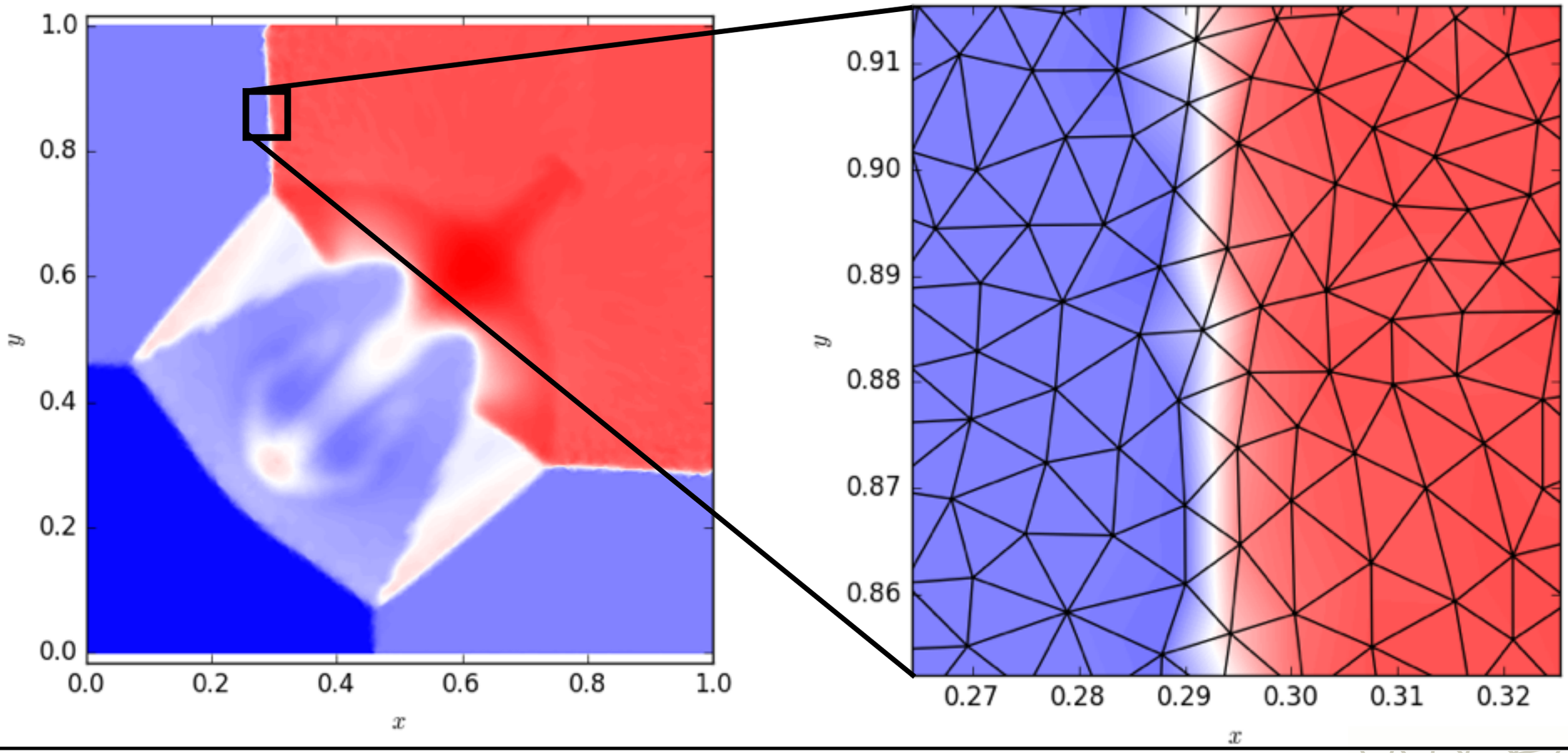
Riemann problem



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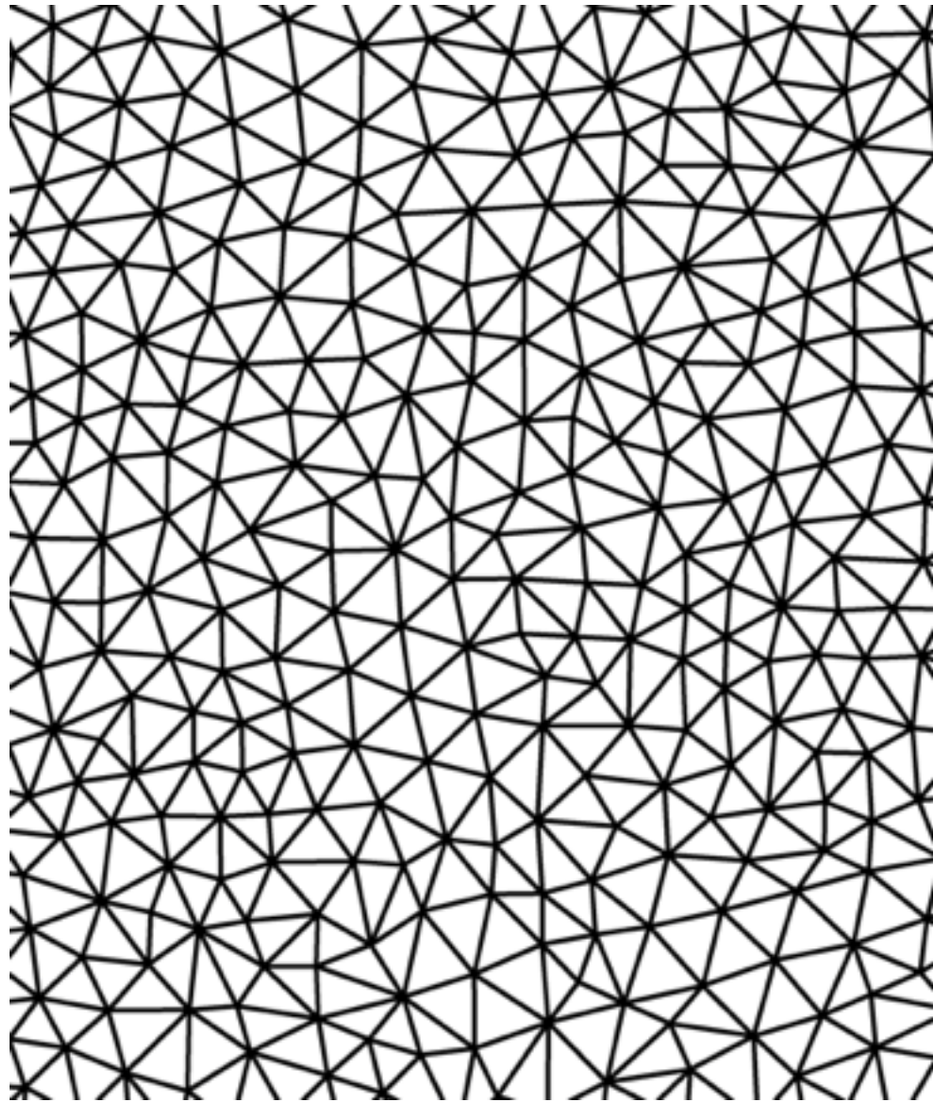
GPU implementation

- Nvidia CUDA
- Unstructured grids: difficult memory access patterns
- Less of a problem for modern GPUs
- Grid generation most difficult



GPU implementation

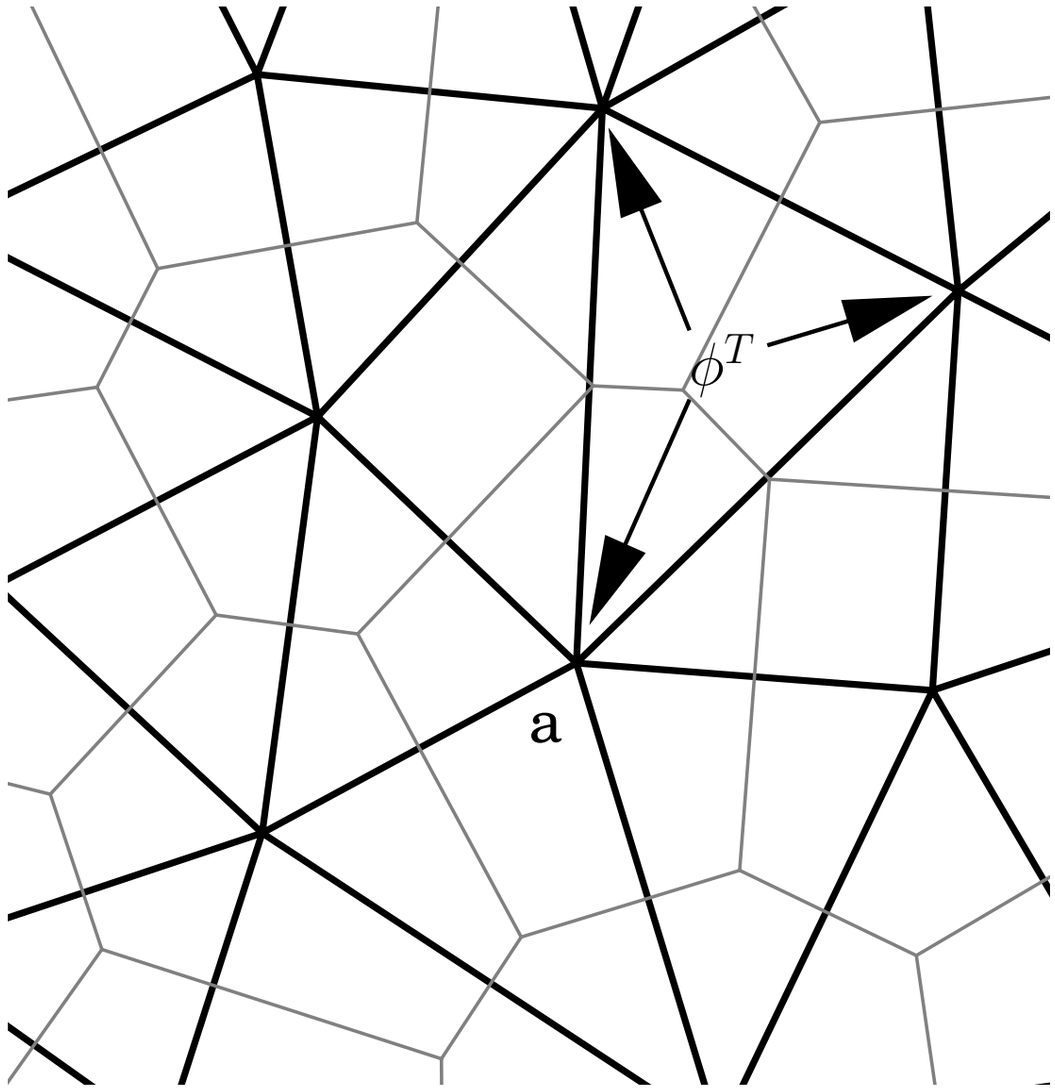
- CPU: Intel Xeon 2 GHz,
  GPU: Tesla K20m

- Speedup**: 100x for grid generation, 250x for hydro

- Limited by low compute-intensity kernels

GPU implementation

- Computing the residual: speedup of 500x
- Distributing: 40x

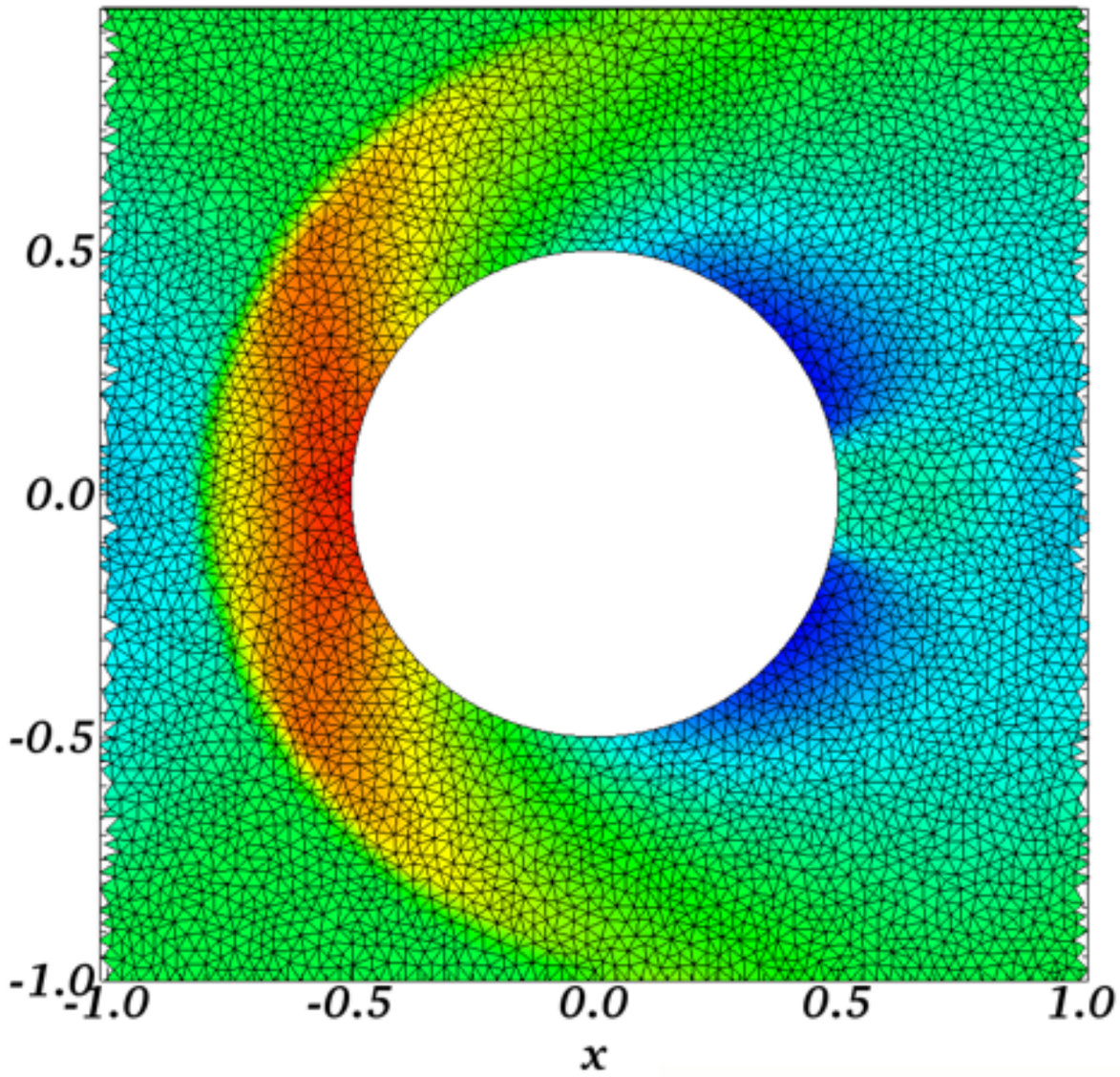


Conclusions

- ASTRIX: a GPU implementation of a multidimensional upwind method on an unstructured grid
- Outperforms Roe solver in many cases
- Open source on GitHub:
  https://github.com/SijmeJan/Astrix

Future

- Adaptive resolution
- Cylindrical coordinates
- 3D/ self-gravity / radiative transfer



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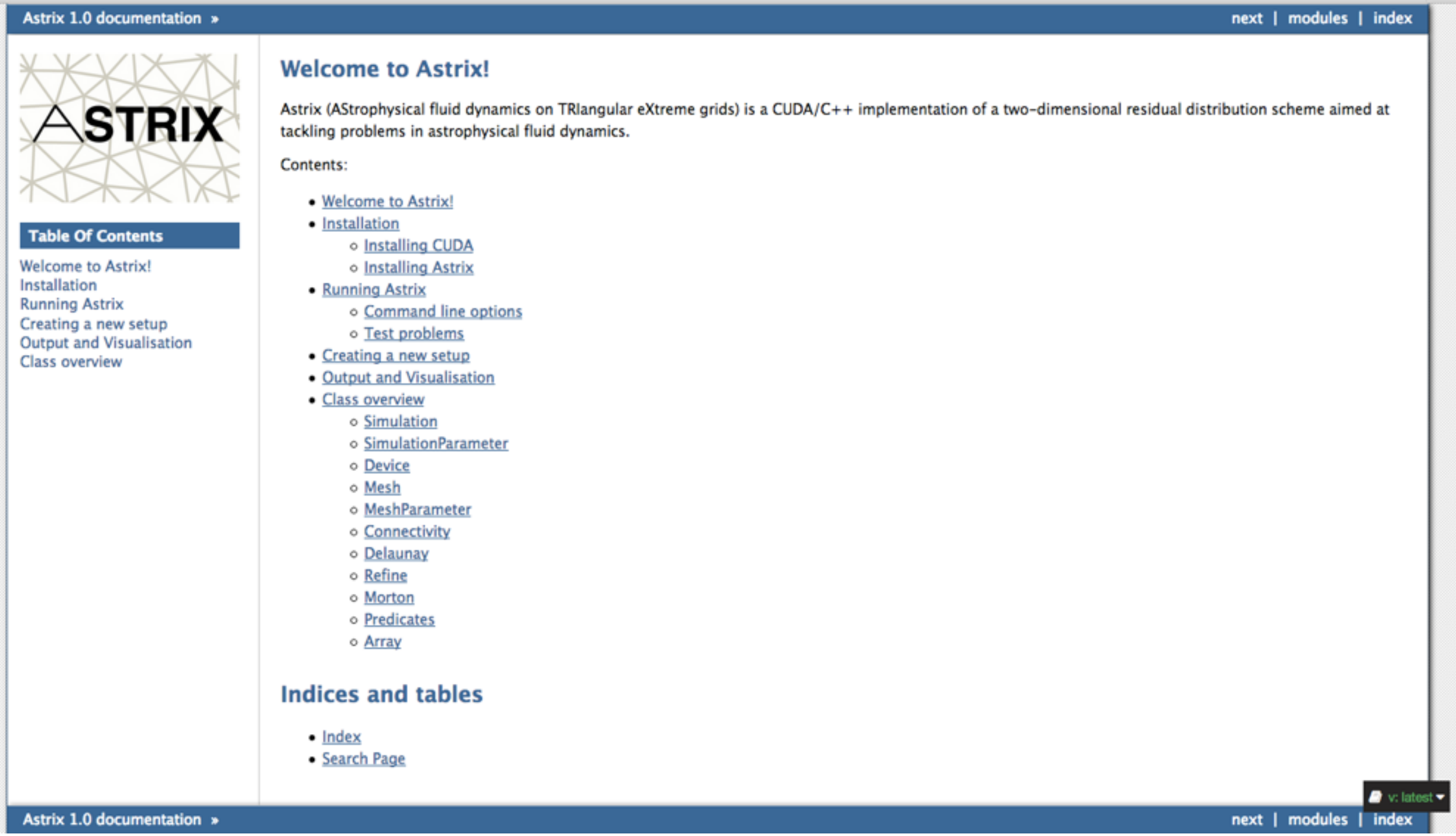

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