Gaining A Dimension: Breaking The Link Between Spatial And Temporal Refinement



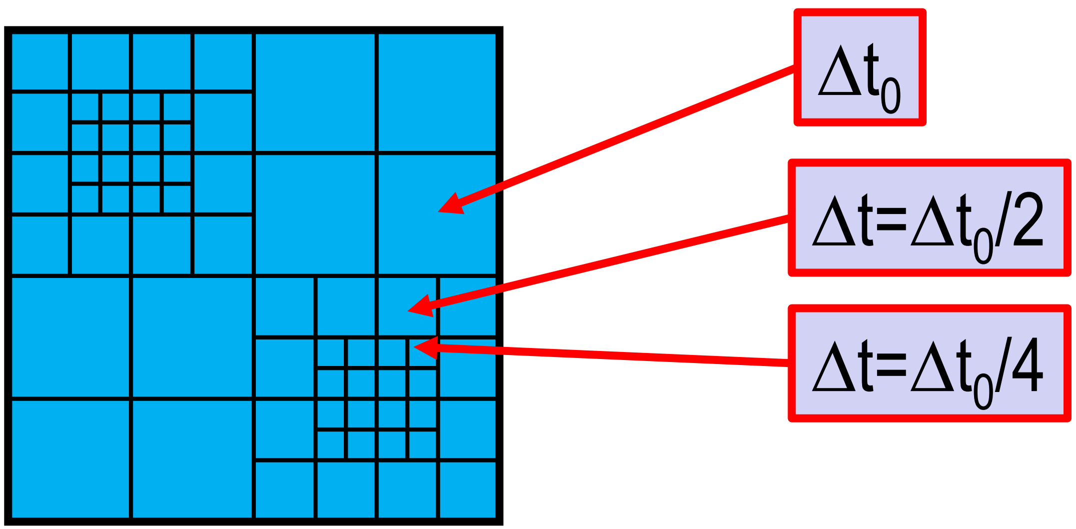
Nick Gnedin

Andrey Kravtsov

Vadim Semenov

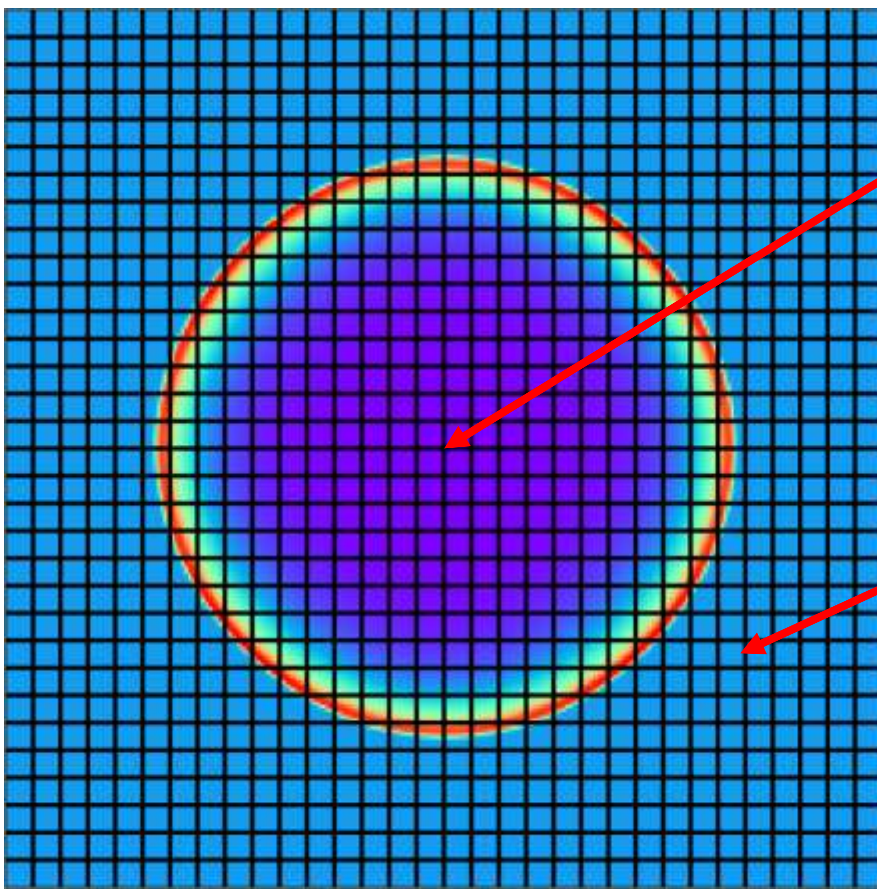
Adaptive Mesh Refinement

- Most typical use of AMR:
  - Refinement in space is flexible (I decide where to refine).
  - Refinement is time is not – it is fully tied to spatial refinement.



Adaptive Time Refinement?

- Special case: no refinement – uniform timestep.



Sets the global timestep.

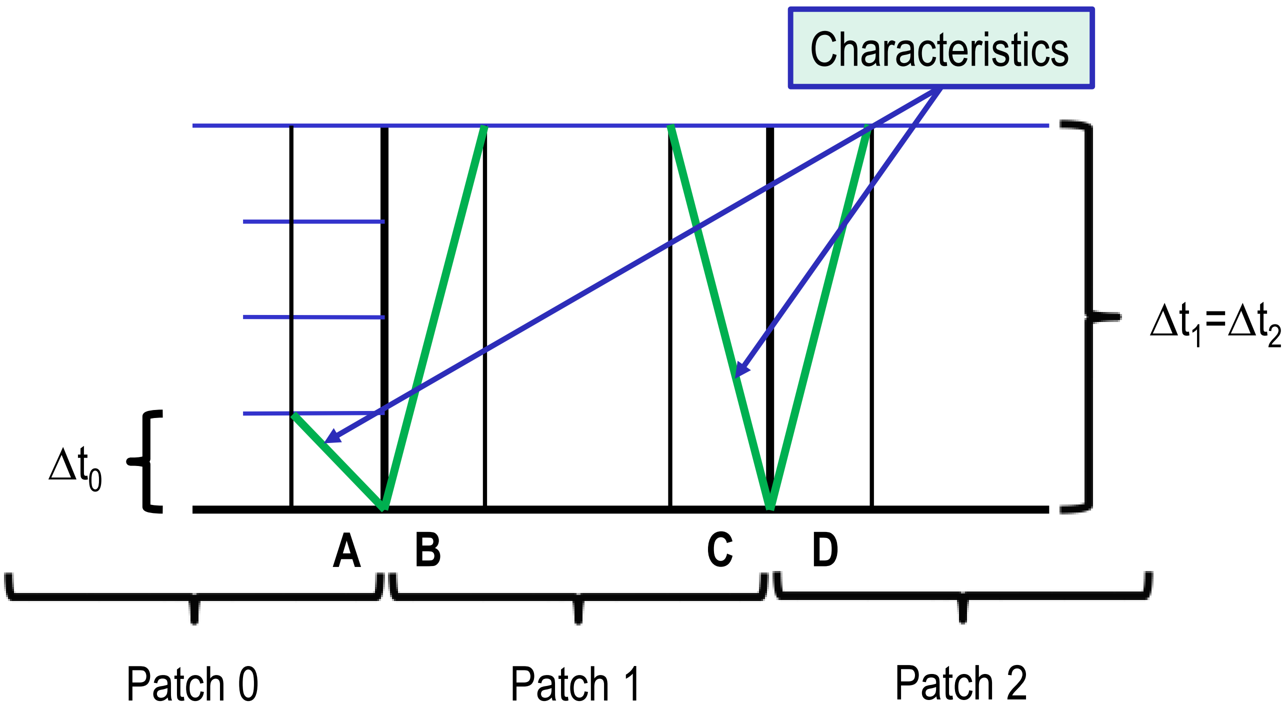
Local CFL condition here is way less strict.

Adaptive Time Refinement?

- Special case: no refinement – uniform timestep.
- With timestepping tied to spatial refinement (either uniform or graded), one cell’s CFL condition sets timestepping for the whole grid.
- One SN explosition in one GMC in a single galaxy may dictate how to timestep to the rest of the (virtual) universe.
- *There is no place for such concentration of power in a modern democratic society!*

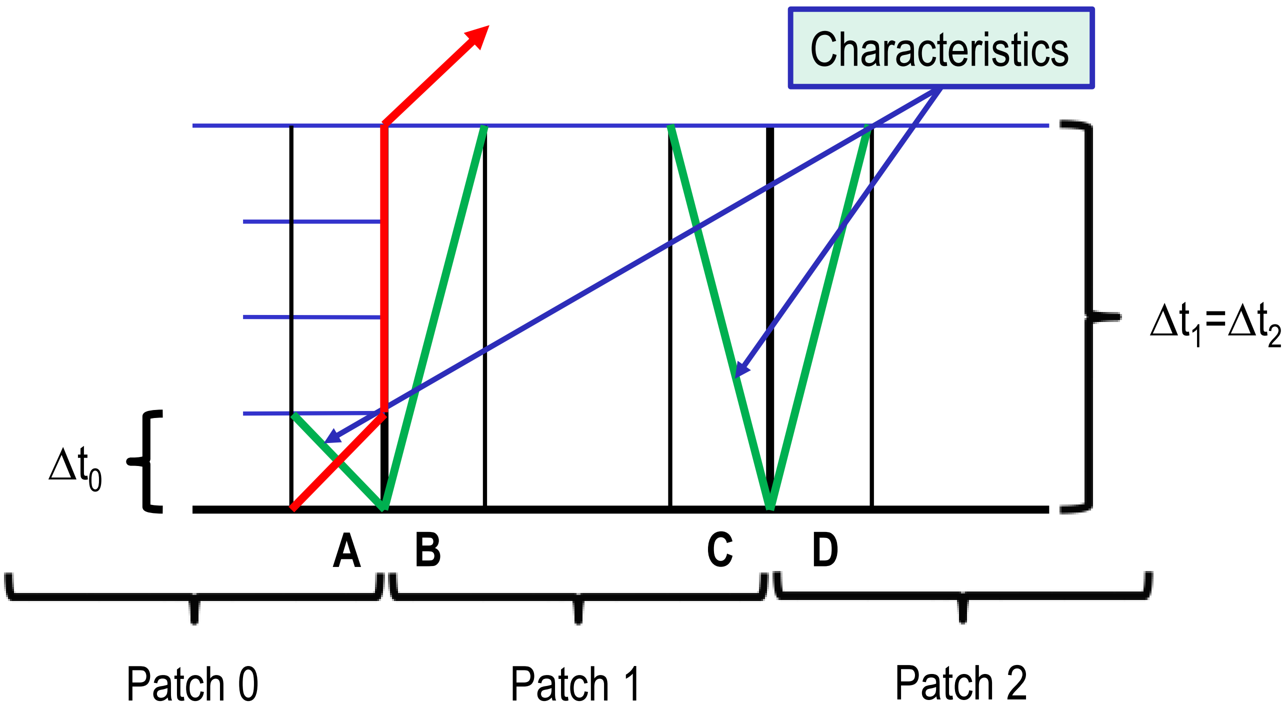
Weak Causality

- Numerical stability only requires the CFL condition to be satisfied in each cell ("weak causality" condition).



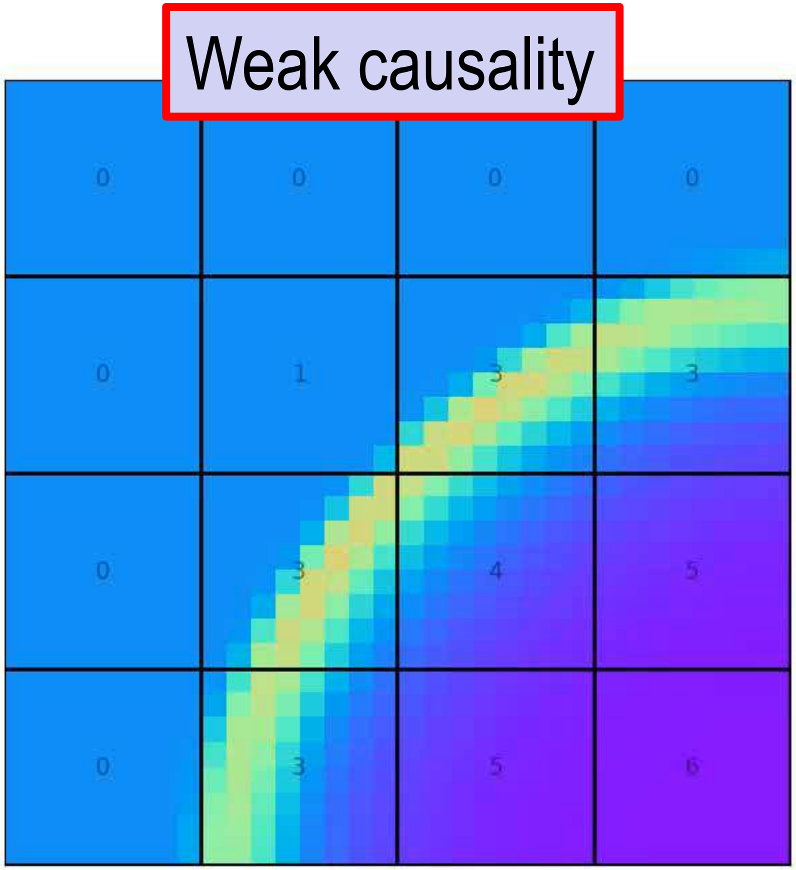
Weak Causality

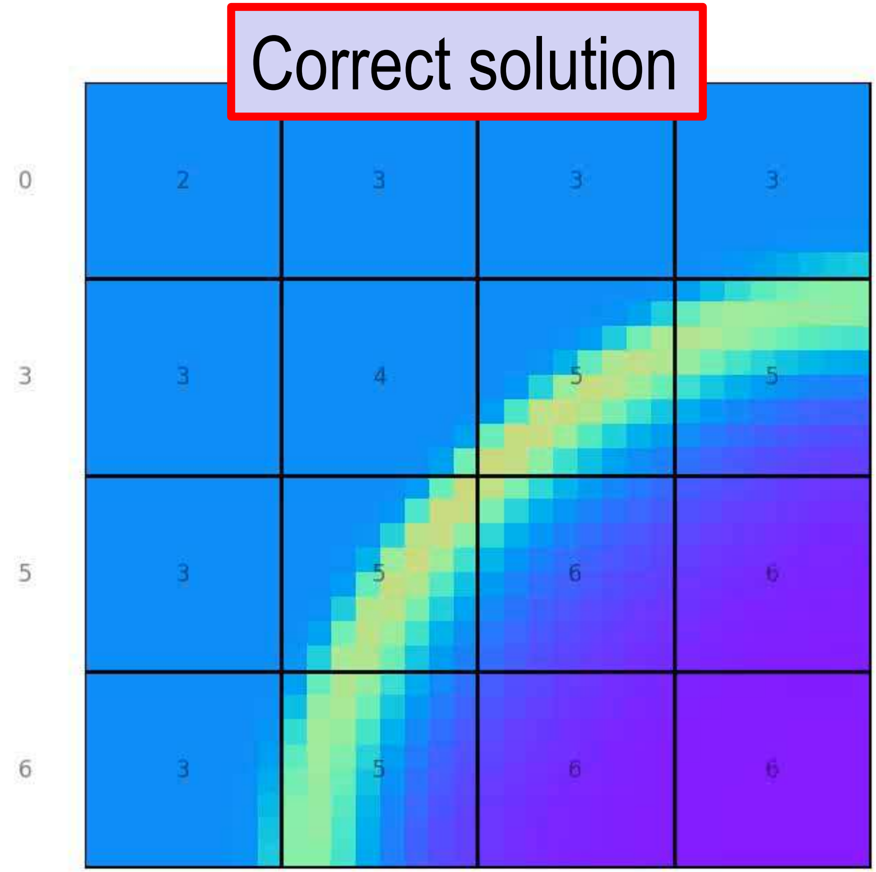
- Under weak causality numerical solution is stable, but not necessarily accurate.



Weak Causality

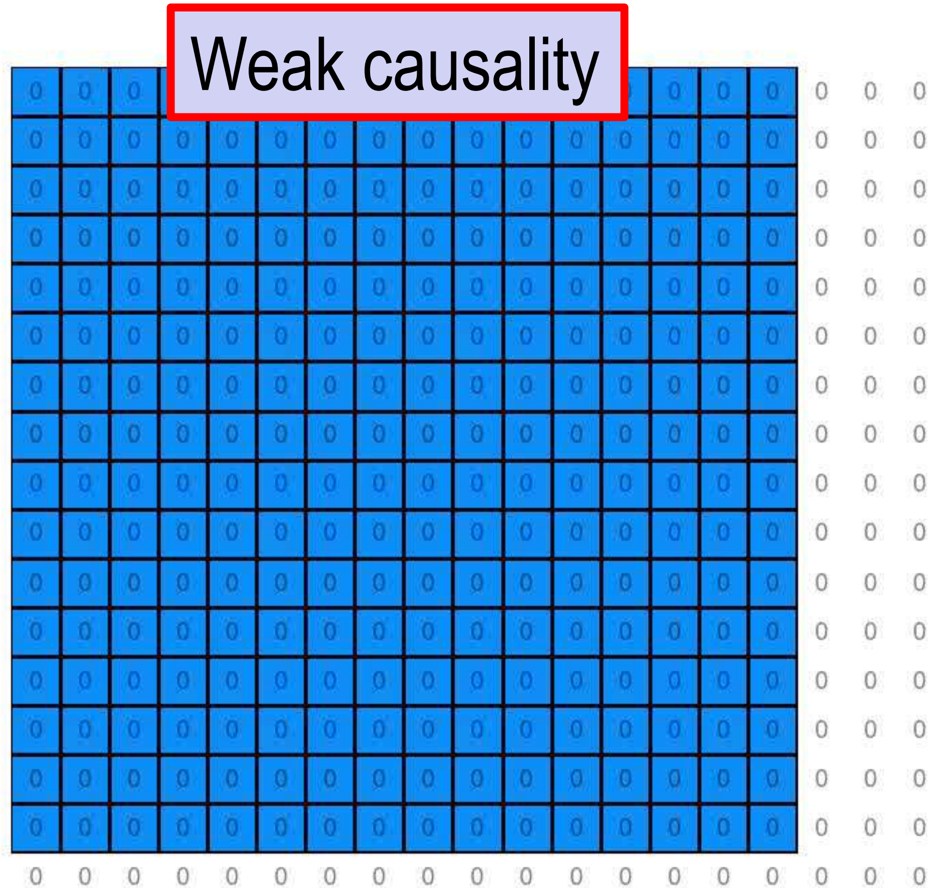
- Weak causality may cause temporary artifacts at timestep boundaries.

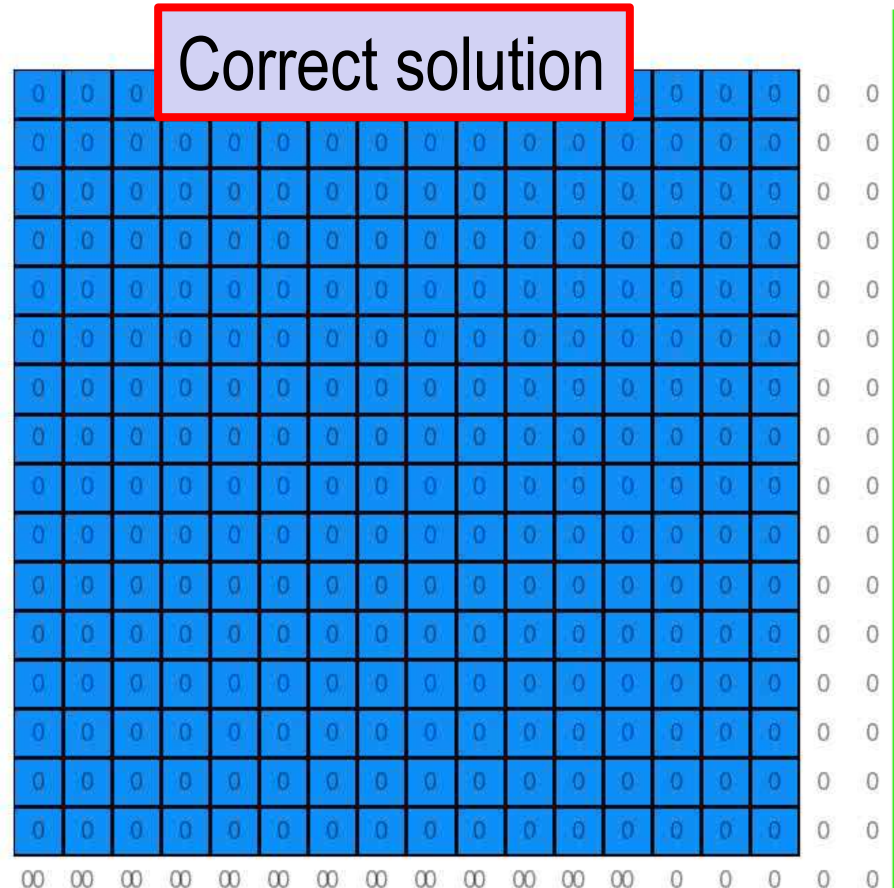




Weak Causality

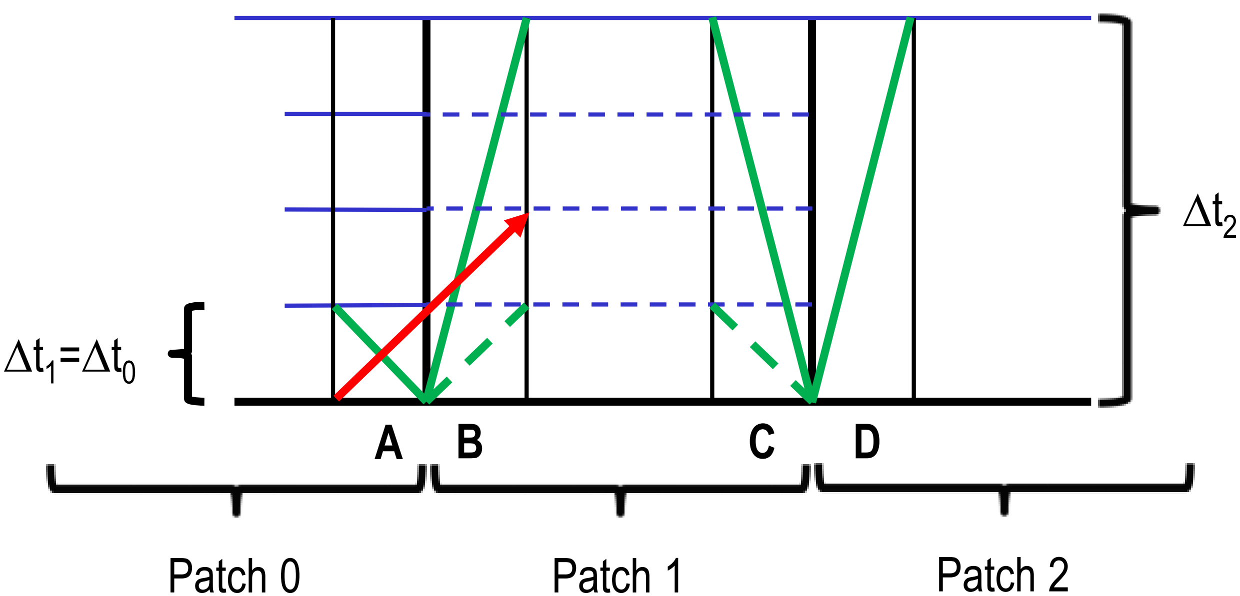
- Artifacts are always at a “cell level” – i.e. hardly noticeable in well resolved flows.





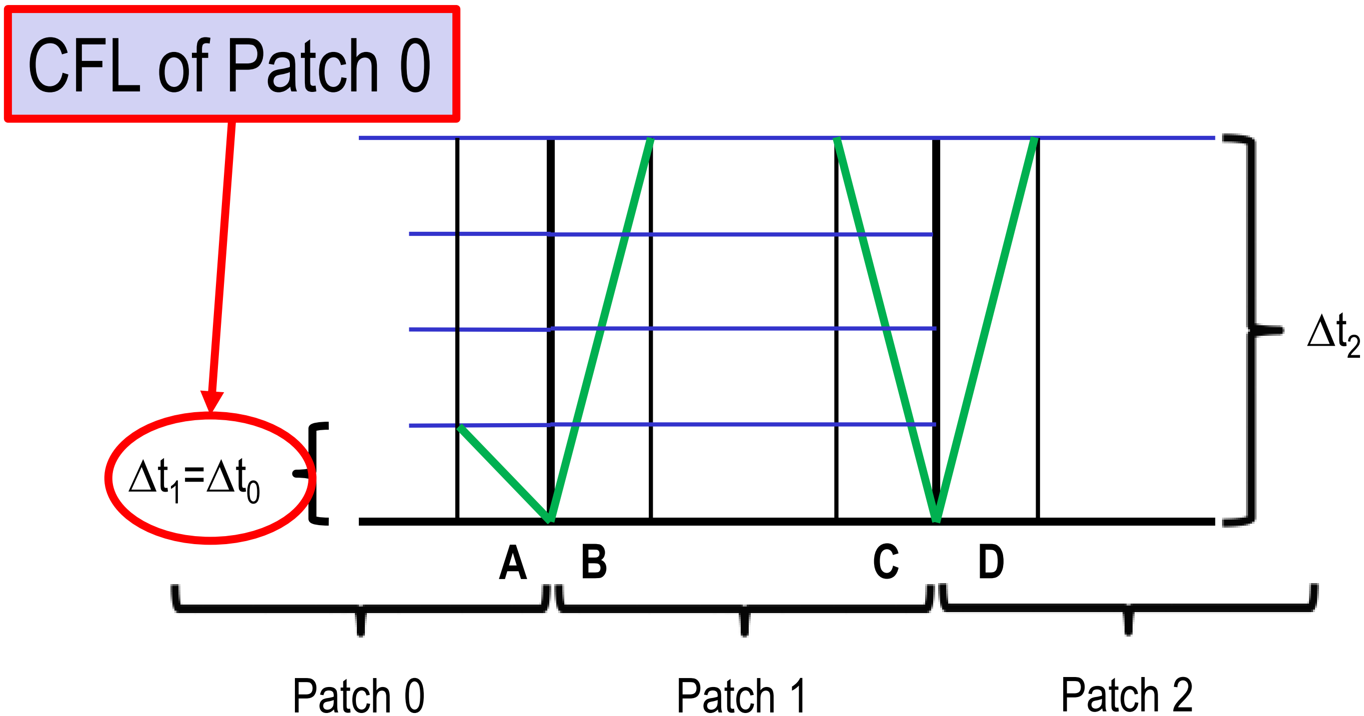
Strong vs Weak Causality

- Artifacts can be eliminated by requiring “strong causality” – that all waves, both linear and nonlinear, propagate correctly.



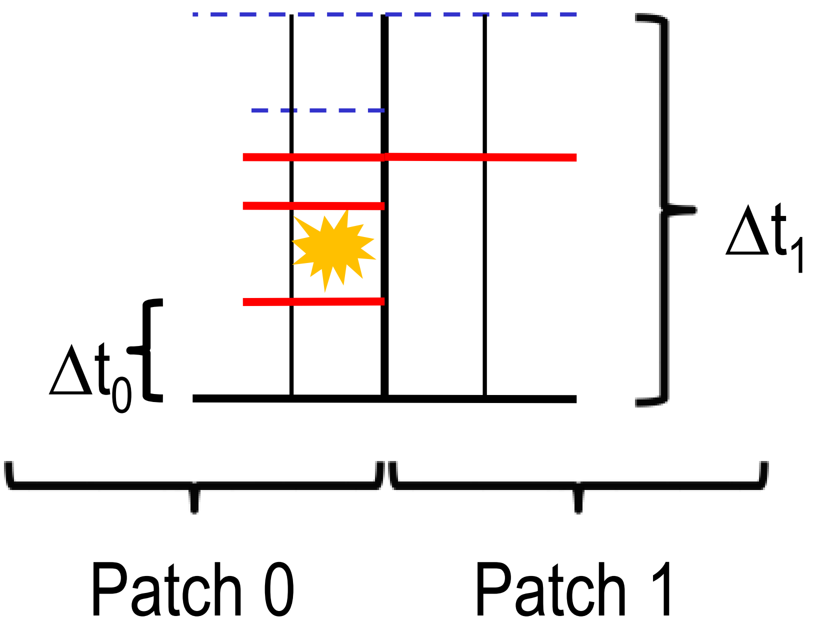
Ensuring Strong Causality: Easy Way

- It is easy to ensure strong causality at a given time: neighbors of patches stepping with their CFL-limited timestep should co-evolve.



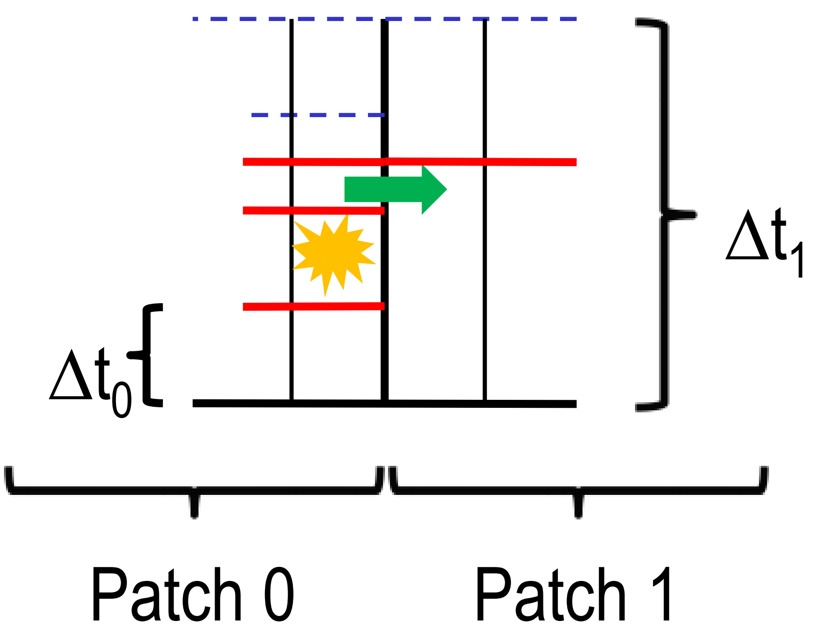
Ensuring Strong Causality: Hard Way

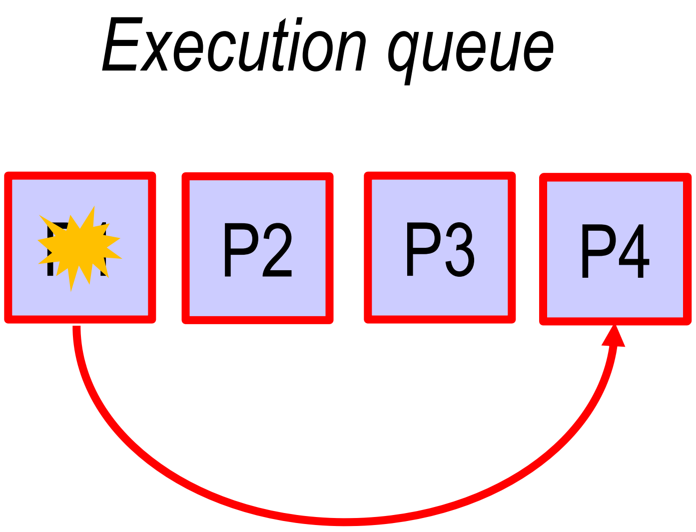
- The easy way does not prevent future CFL violations.
- Timestep for Patch 1 must be modified “mid-flight”.



Ensuring Strong Causality: Hard Way

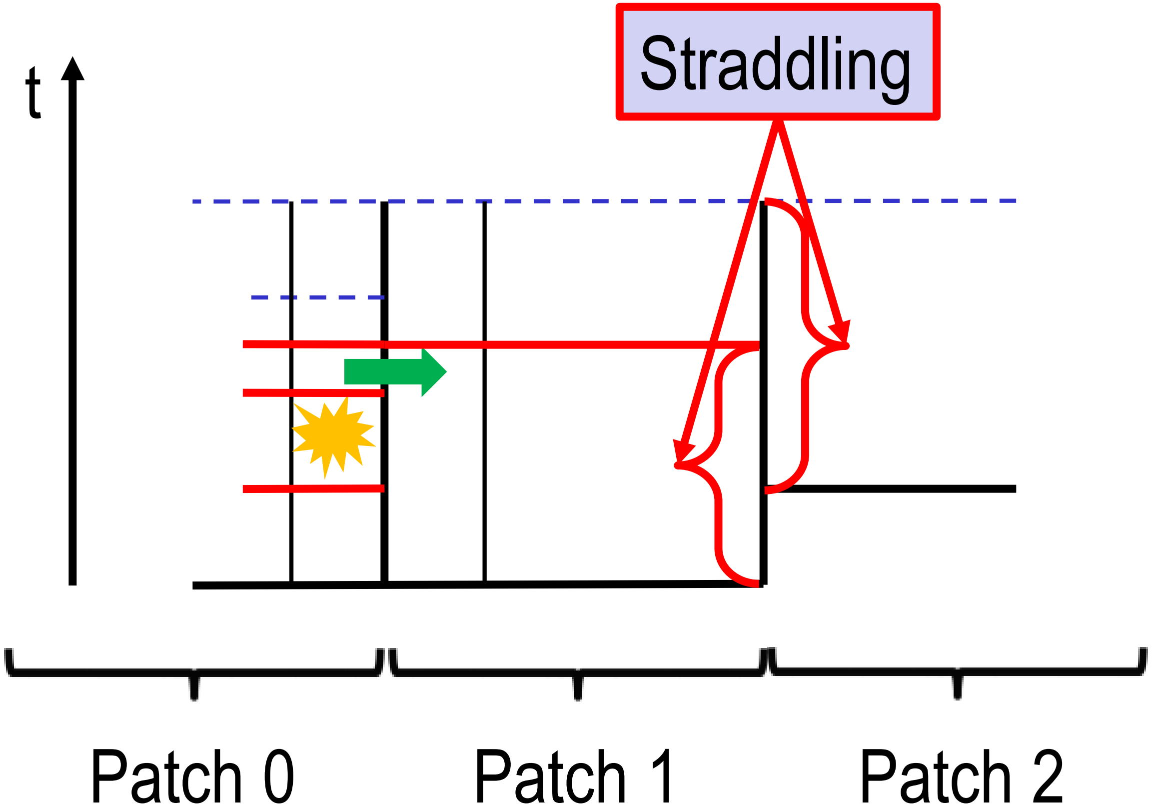
- The sure way of achieving strong causality is one-sided communications.
- That requires an "execution queue".





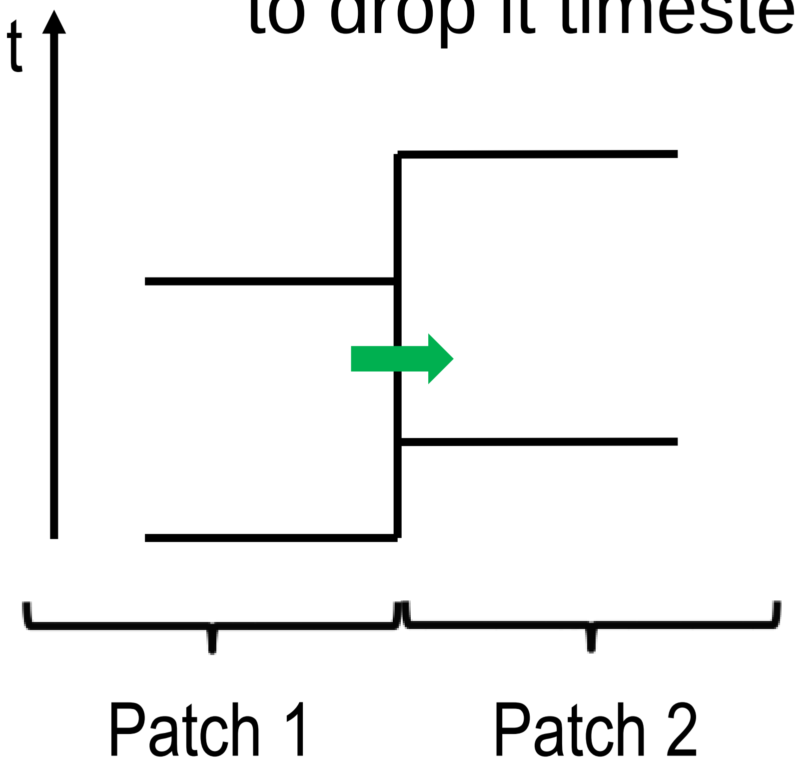
Straddling

- One “message” may not be enough, it may cause “straddling”.



Straddling

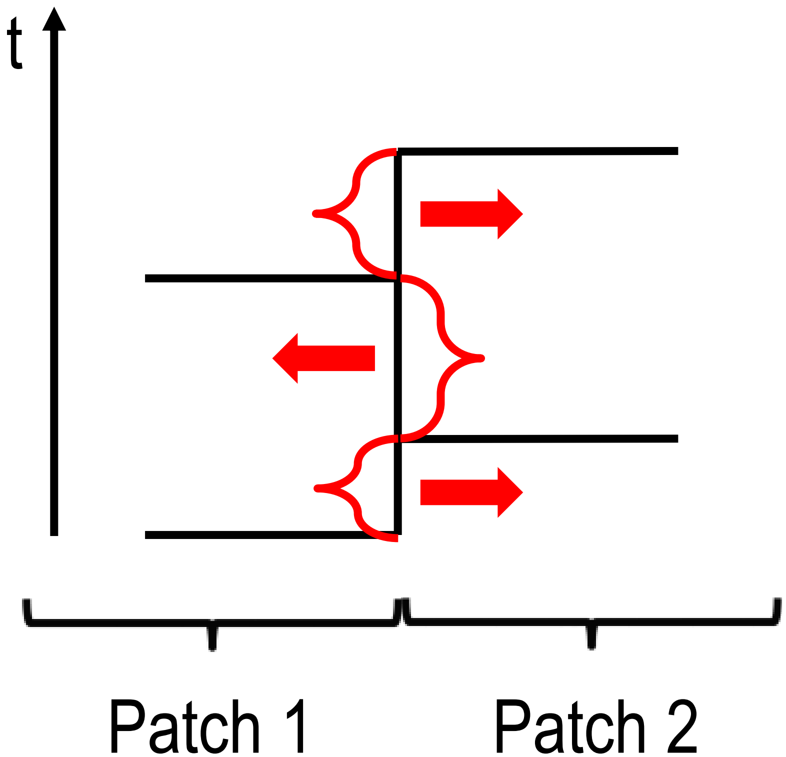
- Two ways to deal with straddling:
  A. Send another message and cause neighbor to drop it timestep.



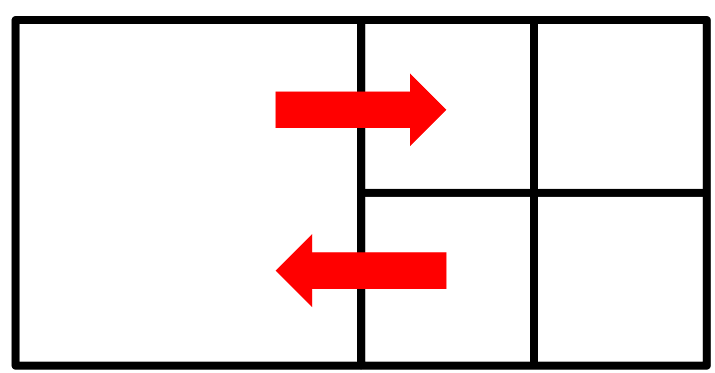
- A wave of such "drops" can propagate over the simulated volume.
- The wave is guaranteed to converge. It may not be easy to detect the end point of the wave in a distributed application.

Straddling

- Two ways to deal with straddling:
  B. Bookkeep who sends flux to whom.

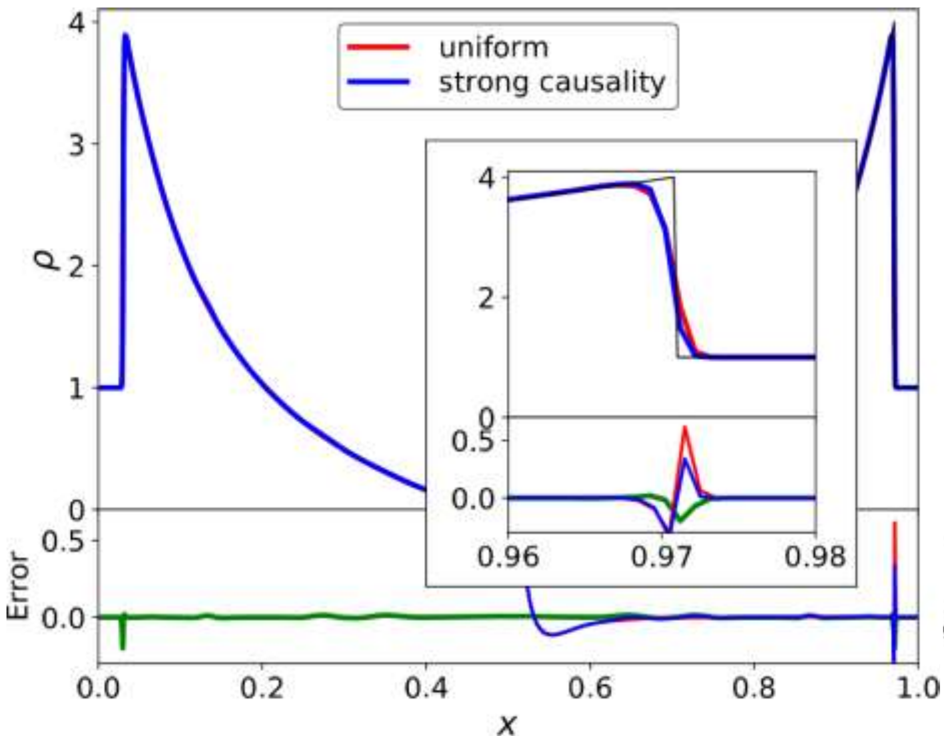


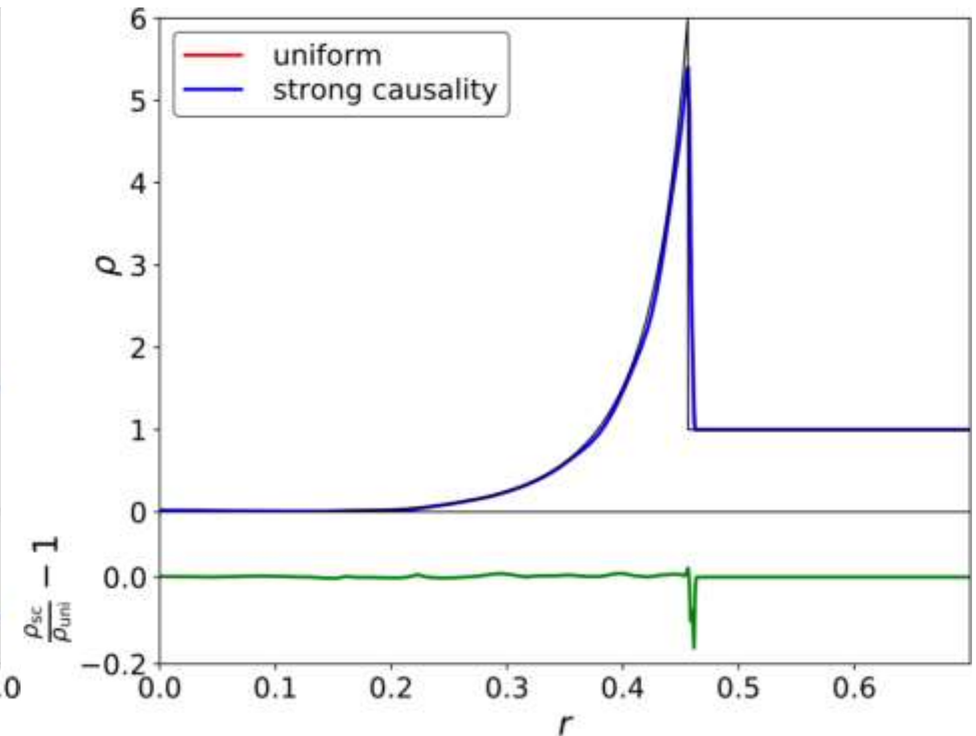
- Bookkeeping is complex, especially with refinement.



What Do We Gain?

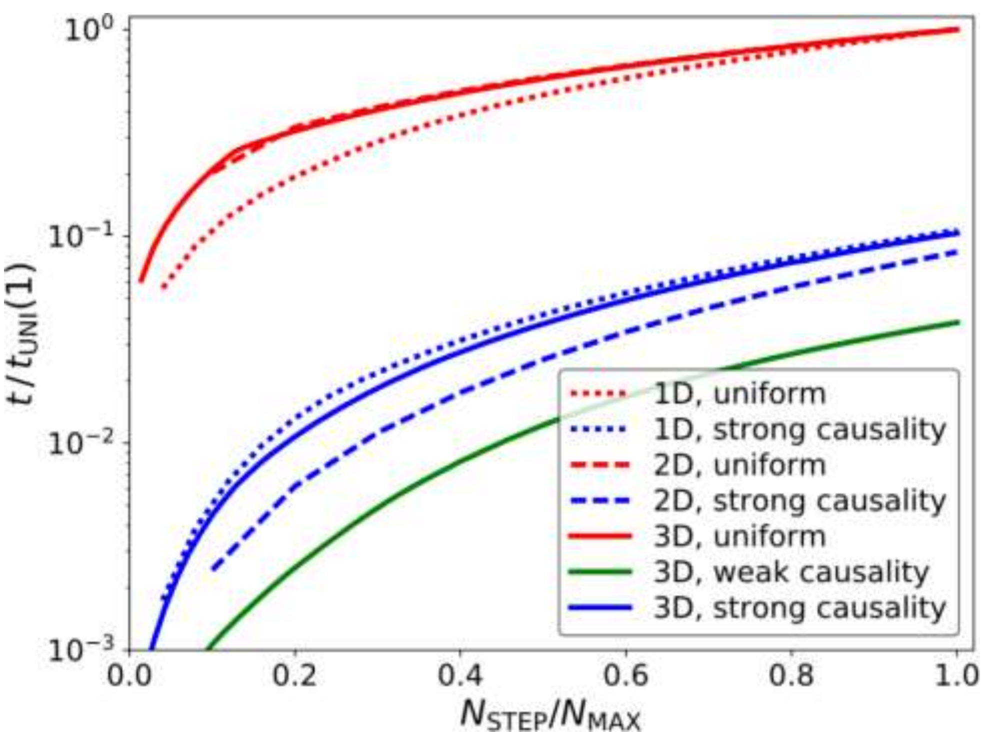
- Simple Sedov blast wave test. Solution with strong causality is up ~10% different from the one with the uniform timestep, but it is actually more accurate!

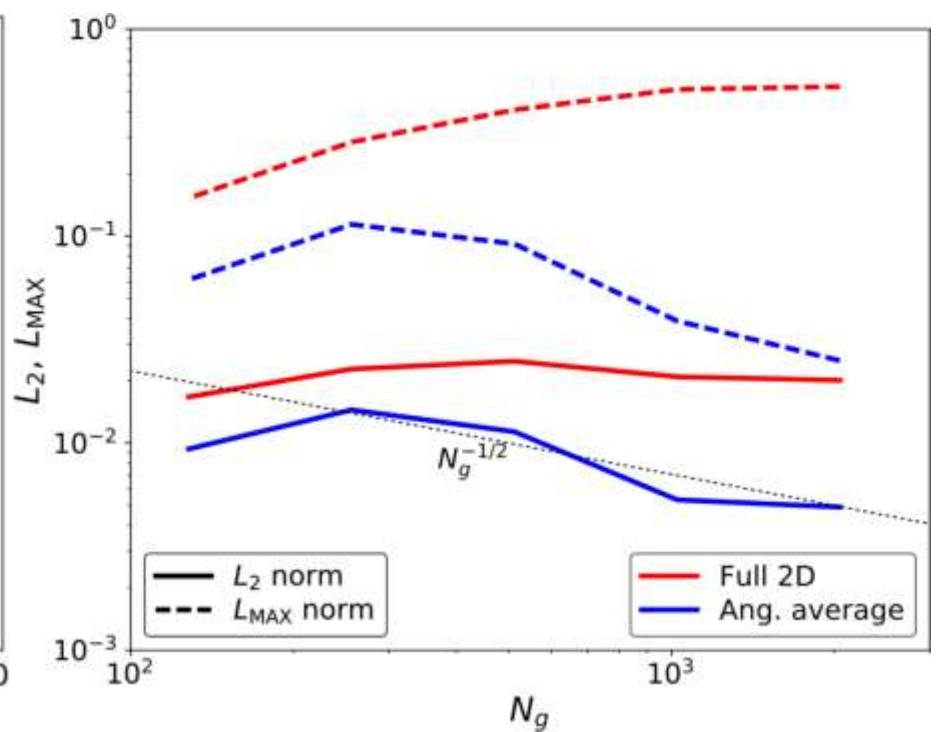




What Do We Gain?

- A variable timestep solution is 10 times faster, but the difference between the two solutions may not disappear in the infinite resolution limit.





Conclusions

- Individual timestepping is a useful numerical technique in many (but, of course, not all) applications.
- Insuring “strong causality” (i.e. correct propagation of all waves) is possible, but may require one-sided communication.
- Solutions with variable timesteps may actually be more accurate than ones with uniform timesteps, since in the advection equation the leading order terms vanish for timesteps with local C=1 (for 1^{st} and 2^{nd} order schemes).