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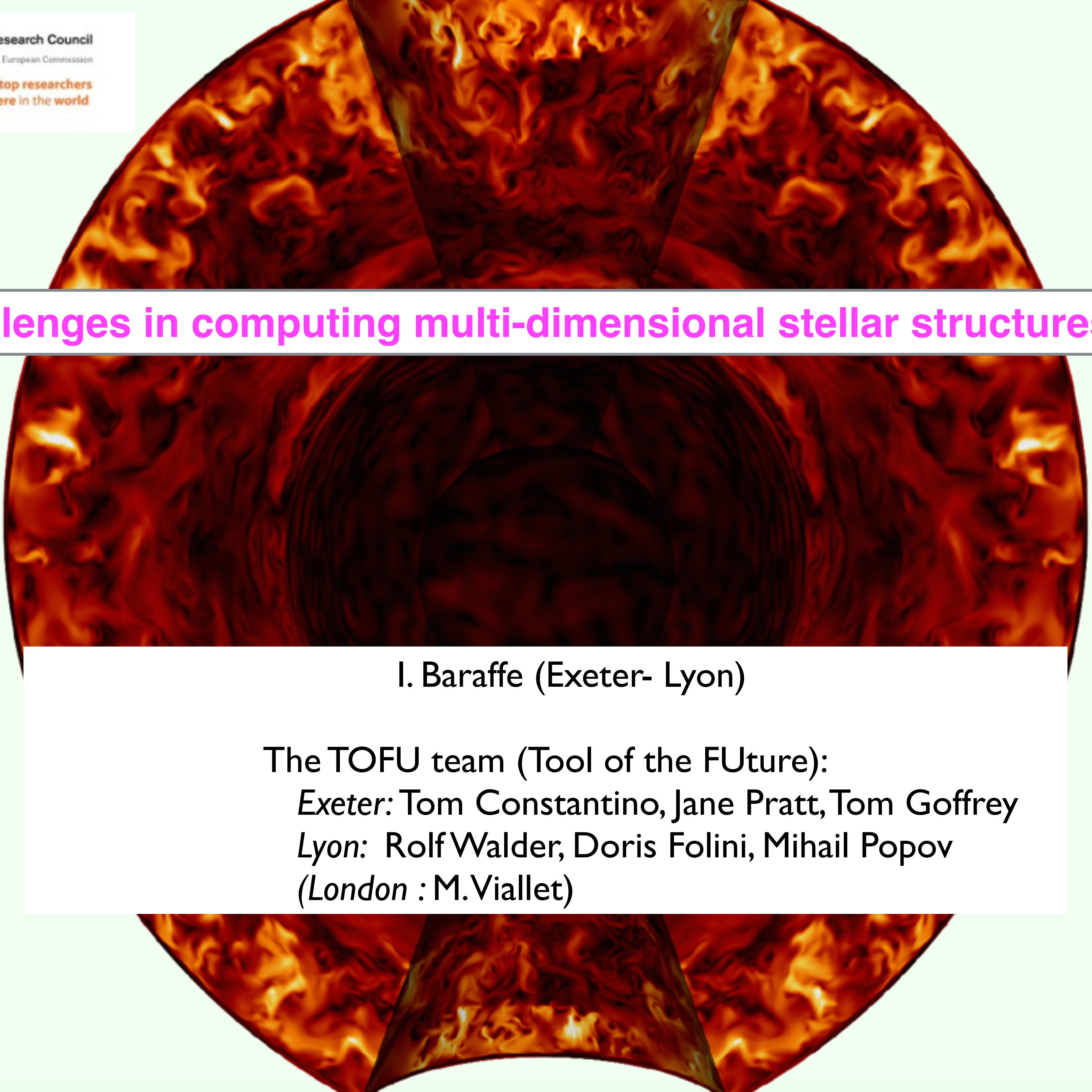

Challenges in computing multi-dimensional stellar structures

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Lyon: Rolf Walder, Doris Folini, Mihail Popov
(London : M. Viallet)

Motivation for a new tool in stellar physics based on time-implicit simulations

Characteristics of stellar interiors:

Many (M)HD processes play key roles on stellar structure and evolution

Convection, rotation, dynamo, mixing, turbulence, etc….

- Characterised by very different timescales

Sun $\tau_{\rm dyn} \sim (R^3/GM)^{1/2}$ ~ 30 min

$\tau_{\rm conv}$ ~ 6 days

$\tau_{\rm thermal} \sim GM^2/(RL) \sim 2\ 10^7$ yr

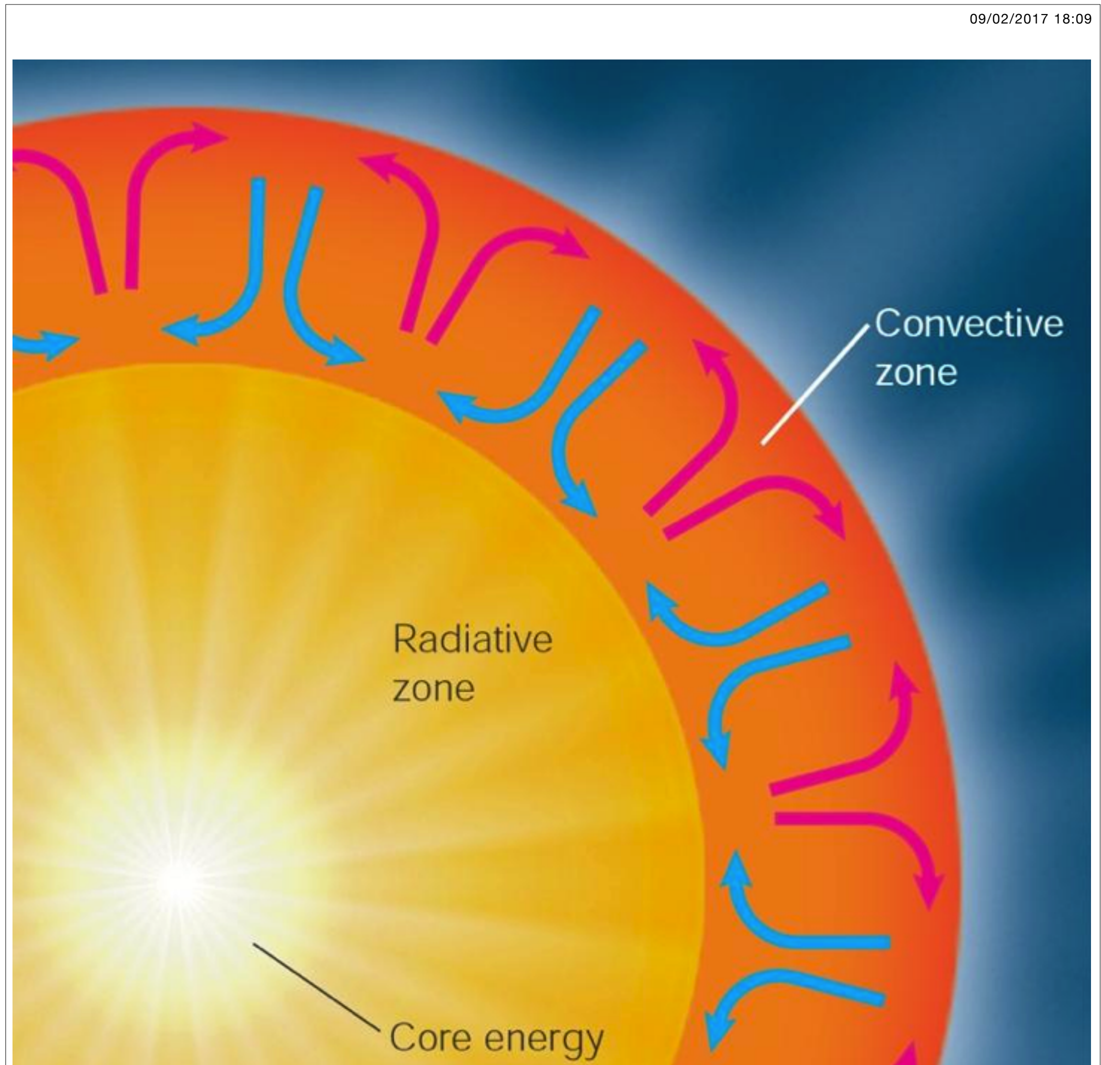
$\tau_{\rm nuc}$ ~ 10^{10} yr

- Very different lengthscales

Pressure scale height: $H_P = dr/d\ln P$

centre: $H_P \sim R_{\rm star}$ Surface: $H_P \sim 10^{-3} - 10^{-2} \times R_{\rm star}$

- Range of Mach numbers ($M \sim 10^{-10}$ - > 1)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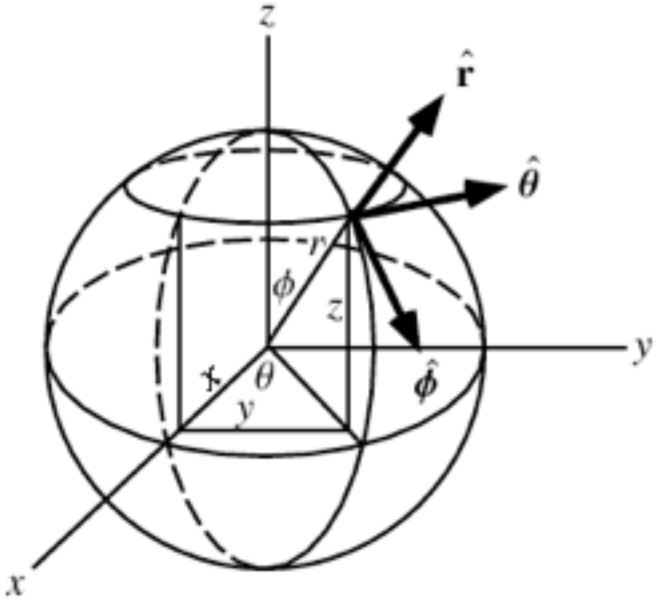
😊 Many successes of 1D (spherical symmetry) models based on phenomenological approaches—> calibration of free parameters from observations

BUT ☹ no predictive power
☹ degeneracy of solutions
☹☹ do we really understand the physics?



⇒ **Need for multi-dimensional models**

(ideally in spherical coordinates)

Motivation 1: 1D Phenomenological approaches have reached their limits

Motivation 2: Need **sophisticated** tools and models to match high quality data (e.g asteroseismology)

Development of MUSIC “Multidimensionnal Stellar Implicit Code”

(Viallet et al. 2011, 2013, 2016; Geroux et al. 2016; Pratt et al. 2016; ***Goffrey*** *et al. 2016)*

- Spherical geometry (2D or 3D)
- Fully compressible hydrodynamics

density

$$\frac{\partial}{\partial t}\rho = -\nabla\cdot(\rho\boldsymbol{u})$$

momentum

$$\frac{\partial}{\partial t}\rho\boldsymbol{u} = -\nabla\cdot(\rho\boldsymbol{u}\boldsymbol{u}) - \overset{\text{gas pressure}}{\nabla p} + \overset{\text{gravity}}{\rho\boldsymbol{g}}$$

internal energy

$$\frac{\partial}{\partial t}\rho e = -\nabla\cdot(\rho e\boldsymbol{u}) + p\cdot\boldsymbol{u} + \nabla\cdot(\overset{\text{thermal conductivity}}{\chi}\nabla\underset{\text{temperature}}{T}).$$

With the radiative conductivity $\chi = 16\sigma T^3/3\kappa\rho$

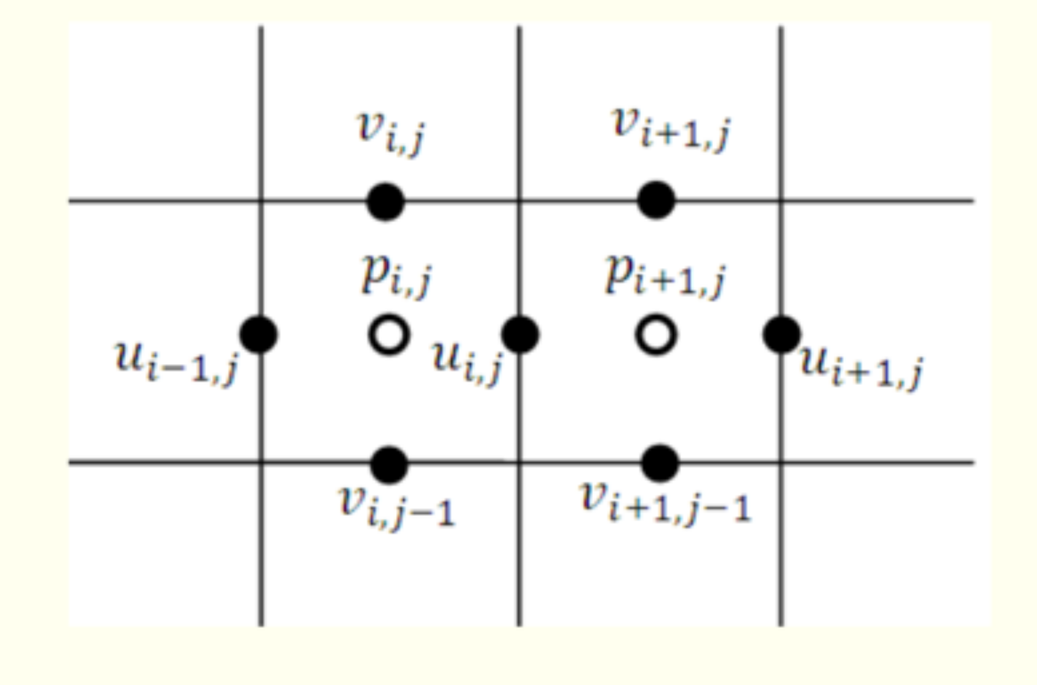
$\boldsymbol{\kappa}$ Rossland mean opacity (OPAL) + realistic equation of state (ionisation, partial degeneracy, etc…)

- Difficulty with various disparate timescales (e.g various stiff scales)

$$\tau_{\mathrm{evol}} = \tau_{\mathrm{therm}}, \tau_{\mathrm{conv}}, \tau_{\mathrm{rot}}, \tau_{\mathrm{nuc}} \gg \tau_{\mathrm{dyn}}$$

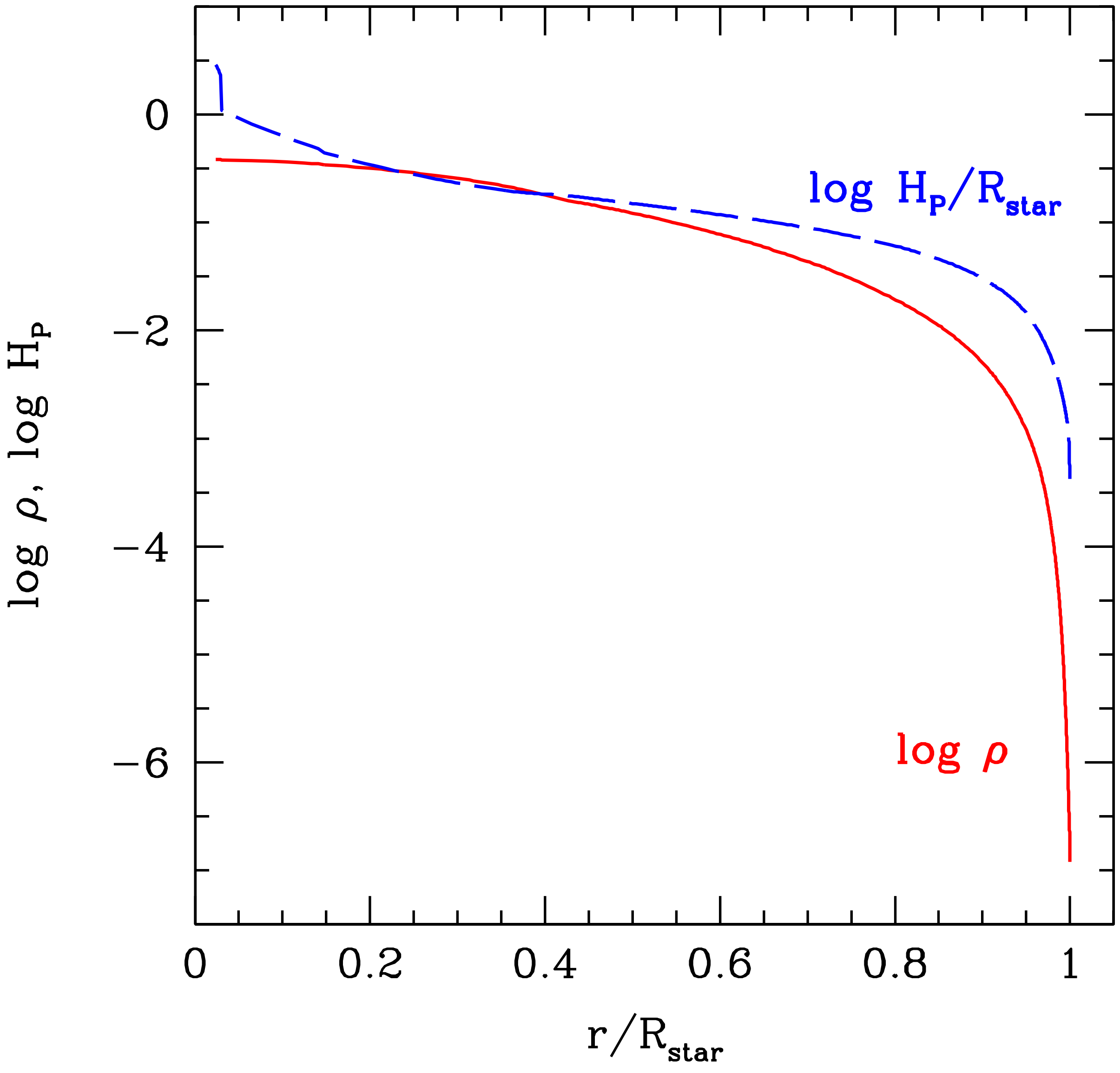
☛ Implicit time integration (see Tom Goffrey's talk)

- Finite volume method on a staggered grid
  *(really **helps for hydrostatic equilibrium*** $\nabla P = -\rho \mathbf{g}$)

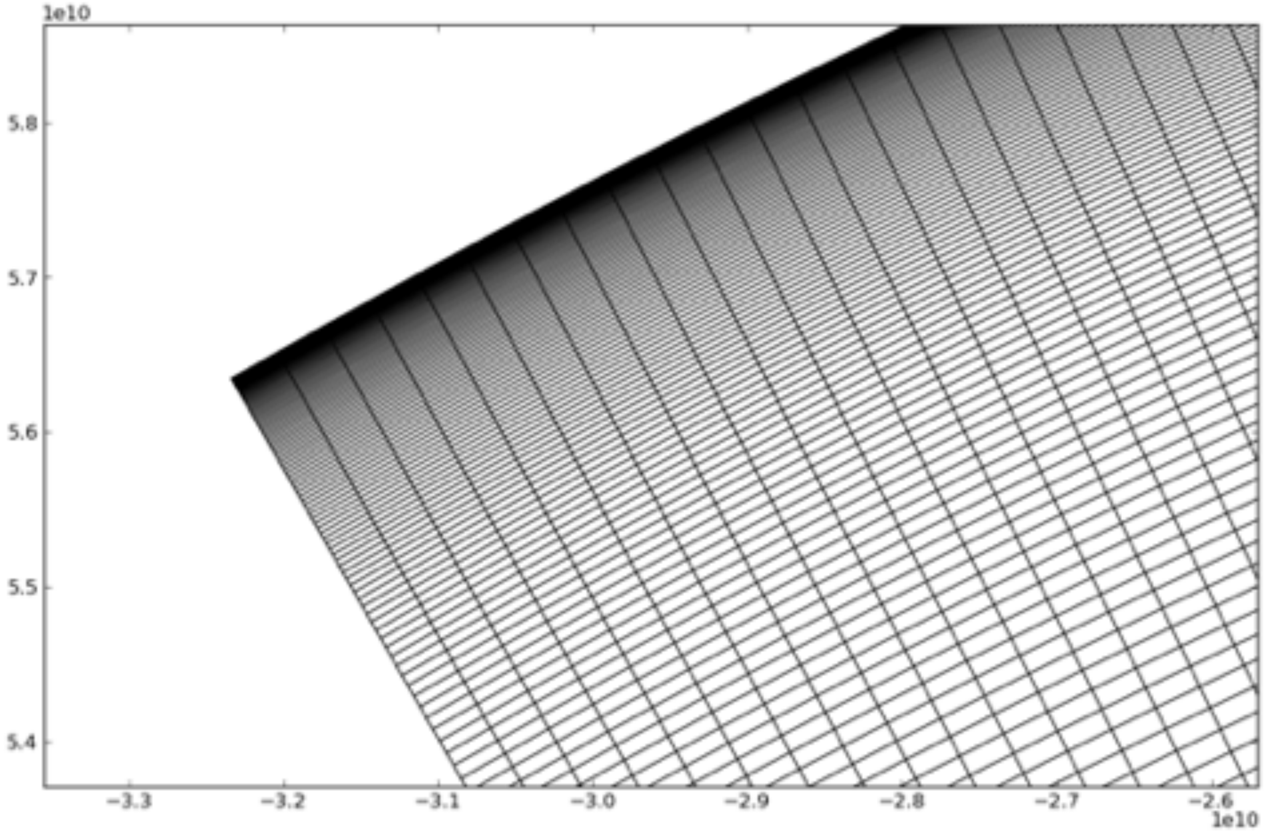


- Initial model from 1D stellar evolution calculation
  ➤ *interface with Lyon code (Baraffe et al.) and MESA (Constantino et al, in prep)*

- **Other specificity (difficulty) characteristic of stellar interiors:**
  - Very different spatial scales from the centre to the surface:
    pressure scale height H_P varies by several orders of magnitude
  - Very steep gradients close to the surface

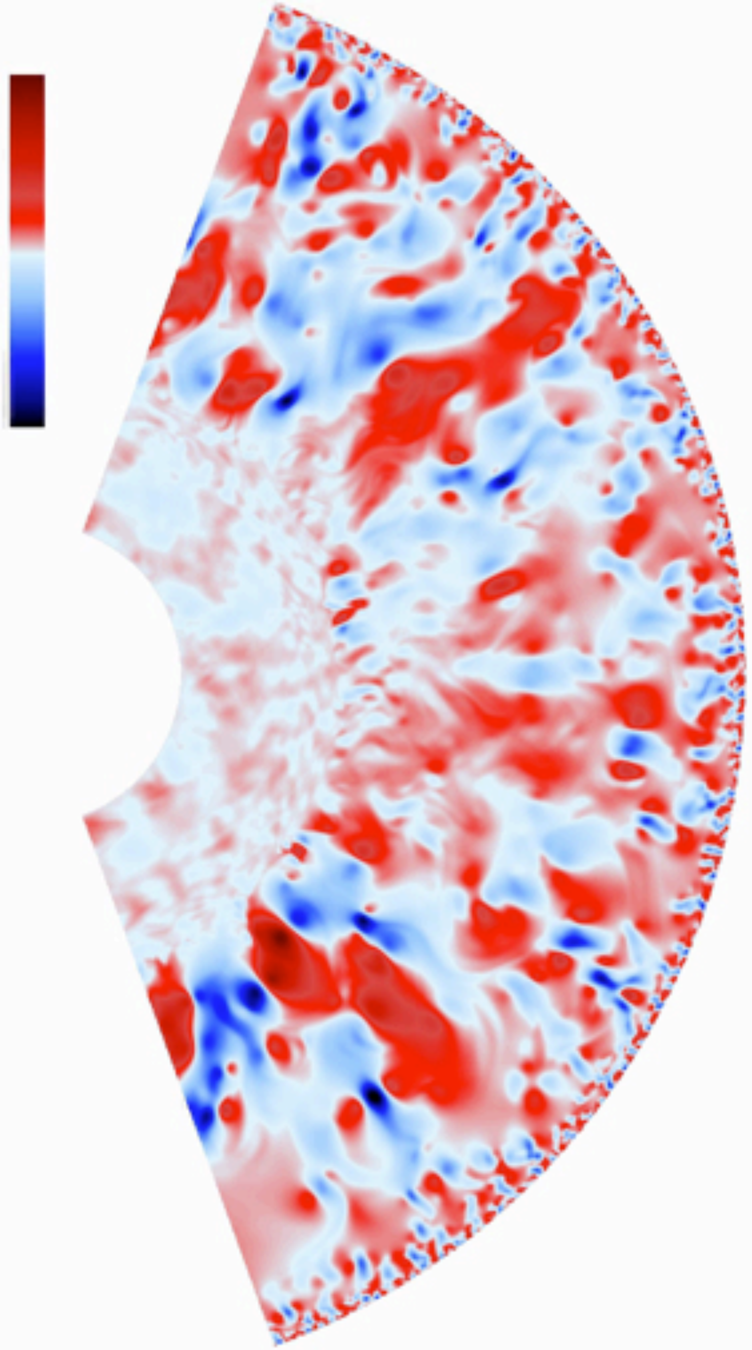


Bad aspect ratio with uniform grid in θ



Current status: Simulations of stars of various masses in 2D/3D slices from central region to surface

Performances for a young (pre main-sequence) star (1 $M_\odot$, ~ 60% convective envelope)



- 2D simulations up to 2432x2048 (from ~ 0.1 R to surface)

  1024^2 with *256 procs, 72hr wallclock time for 1 convective turnover* (*$\tau_{conv} \sim 10^6$ s ~ 10 days*)

- 3D simulations up to 512^3

  256^3 with *512 procs, 6 days wallclock time for one τ_{conv}*

MUSIC is parallelized using MPI domain decomposition

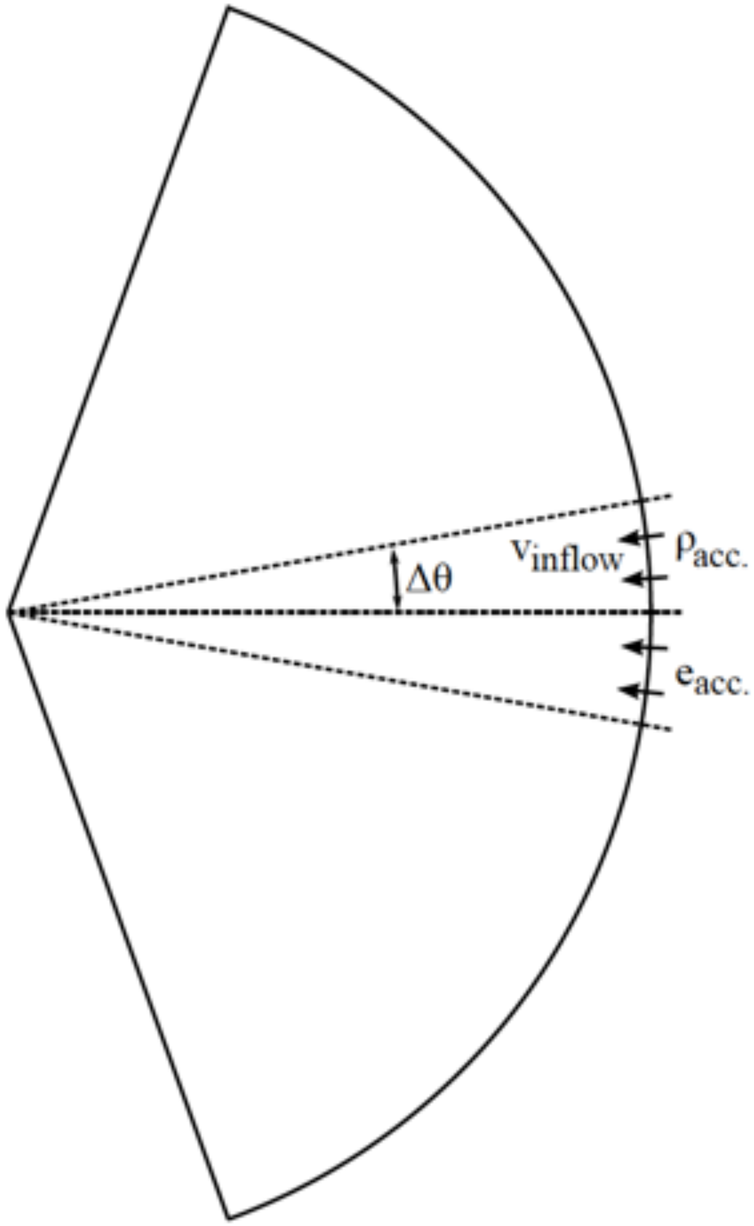
Pratt et al. 2016

➡ **First application:** **Effect of accretion on the structure of very young objects**

(Geroux et al. A&A, 2016)

Test one main assumption in current 1D stellar evolution codes: instantaneous redistribution of accretion **mass** and **energy** in the interior

Effect of "burst" (episodic) accretion **$\dot{M}$= 10^{-4} $M_\odot$/yr** on a young convective star



$$\rho_{\rm acc} = \dot{M}/(A_{\rm acc} v_{\rm inflow})$$

$$A_{\rm acc} = 4\pi R^2 sin(\Delta\theta)$$

A_{acc}, v_{inflow}, $\Delta\theta$ from boundary layer models of Kley & Lin (1996)

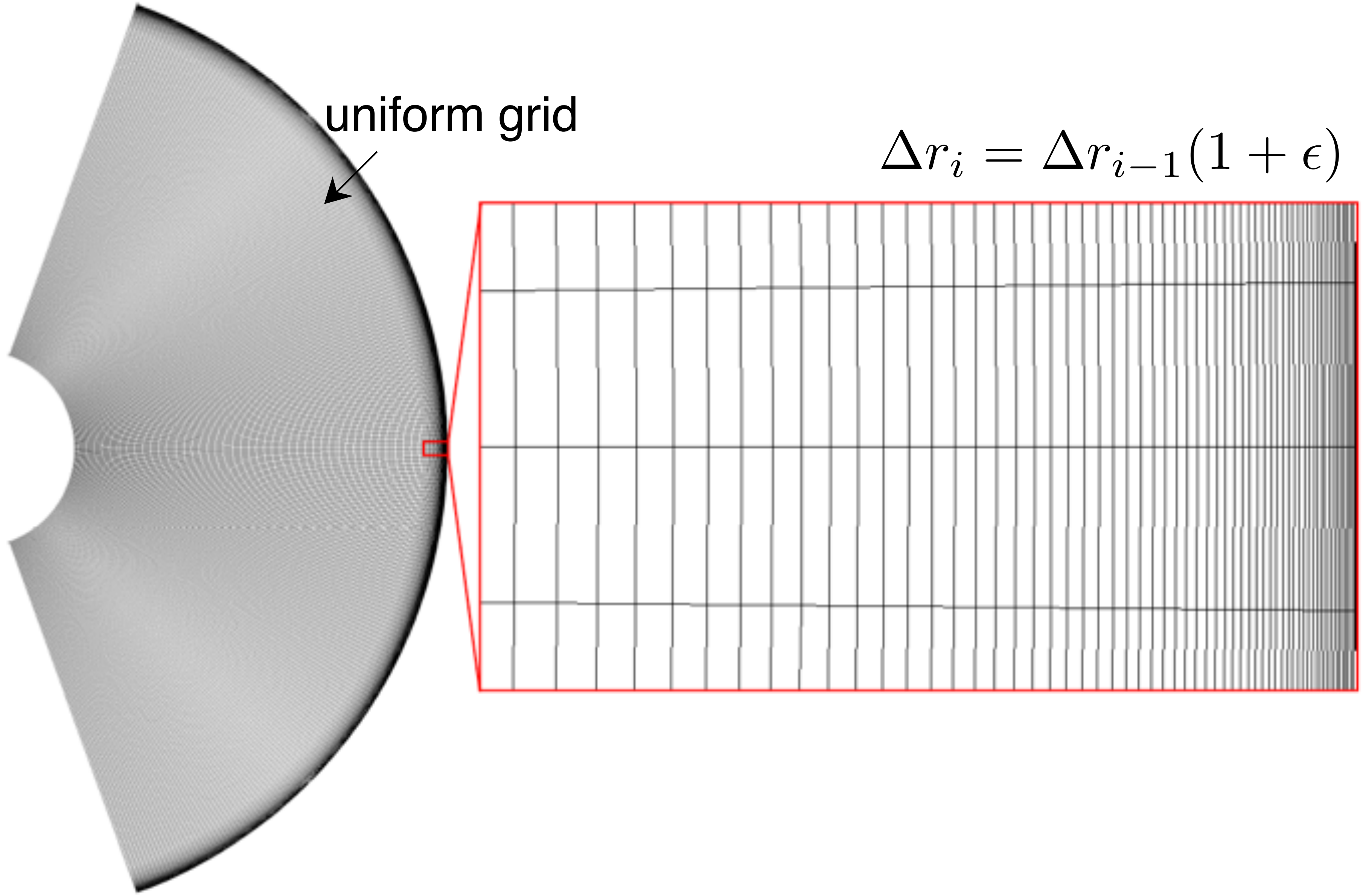
Effect of amount of accretion energy absorbed $L_{acc} = \alpha\, \varepsilon\, (GM\dot{M})/R$

$\alpha \sim 0$ —> “cold” accretion

$\alpha > 0$ —> “hot” accretion

☛ **Treatment of the surface must be realistic with $F_{surf} = \sigma T^4$**

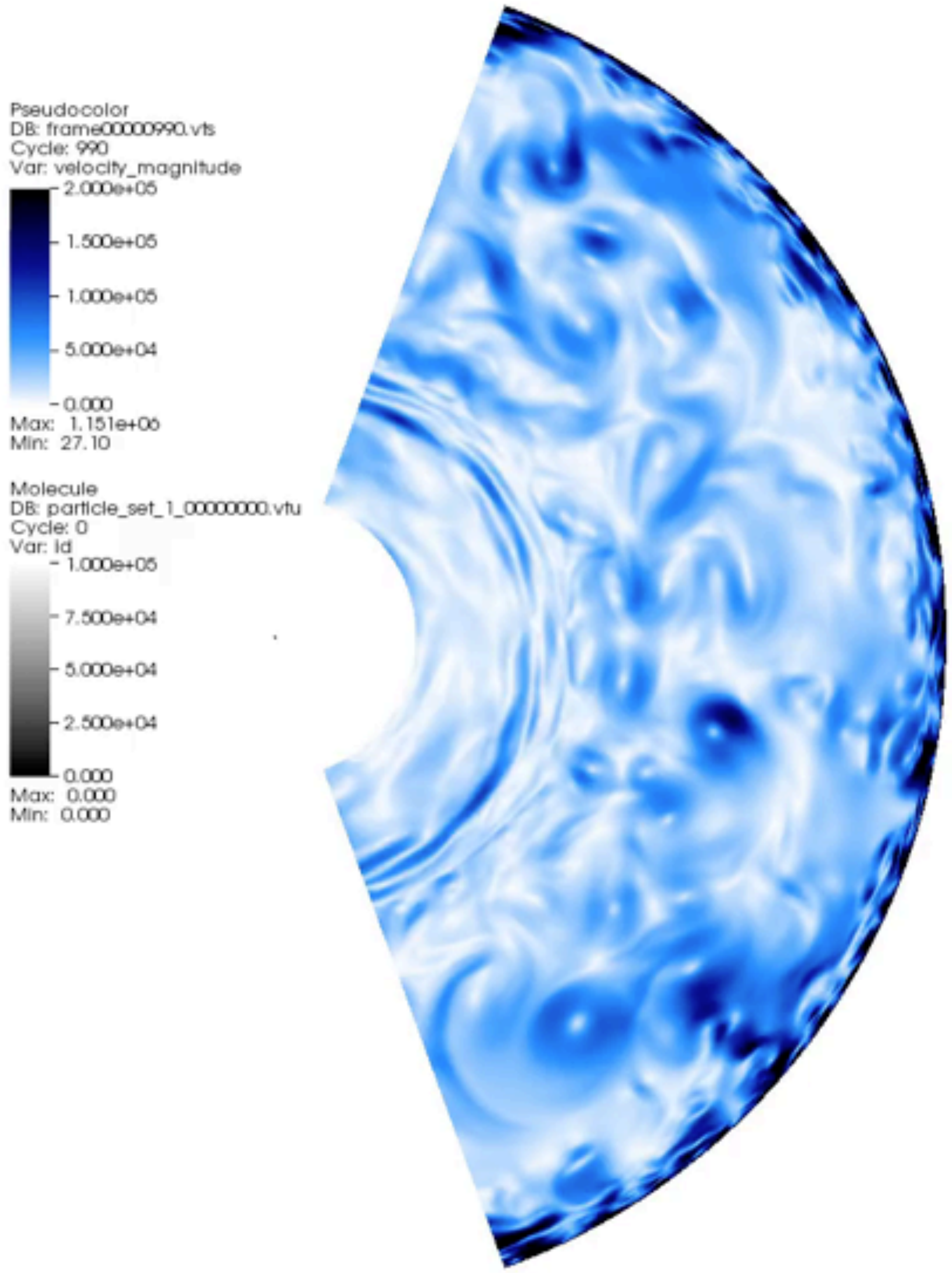
⇒ **Use of a spliced grid to resolve smaller scales/steep gradients**



$$\Delta r_i = \Delta r_{i-1}(1+\epsilon)$$

First result:

Multi-D simulations confirm assumption in 1D codes of instantaneous and homogeneous redistribution of accreted material for cold/warm accretion **for $\alpha \lesssim 0.1$**



$(L_{acc} = \alpha\, \varepsilon\, (GM\dot{M})/R)$

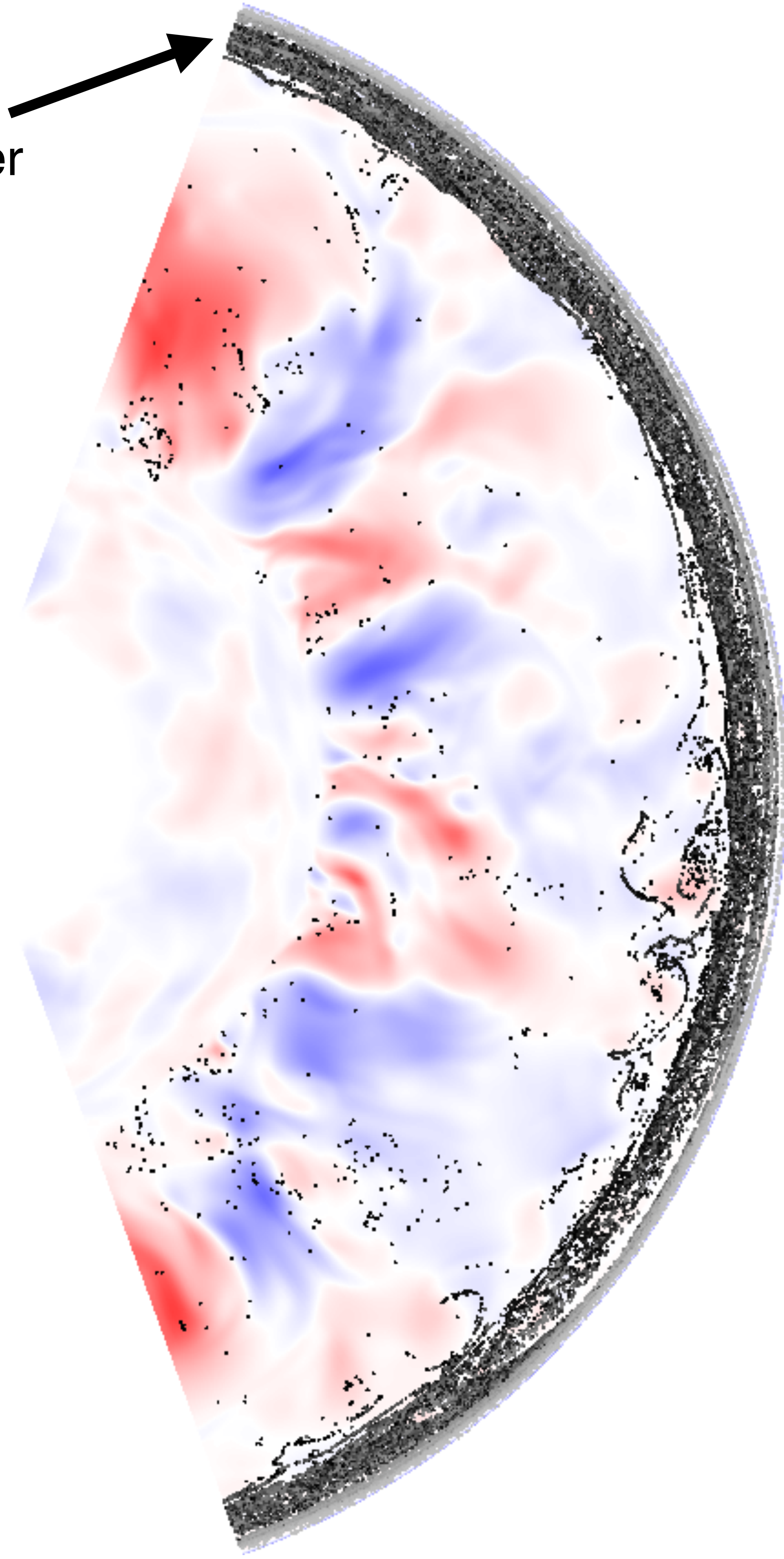
Visualization with tracer particles on velocity field

Second result:

- For hot accretion ($\alpha \gtrsim 0.1$), formation of a hot surface layer (no deep mixing of accretion energy)

- Assumption in 1D codes of redistribution of accretion energy deep in the interior **overestimates the effect** on the structure for $\alpha \gtrsim 0.1$ (expansion of accreting object)

☛ Use of an accretion boundary condition $L_{surf} = L_{acc}$ is more realistic in 1D codes
(see details in Geroux et al. 2016, A&A)

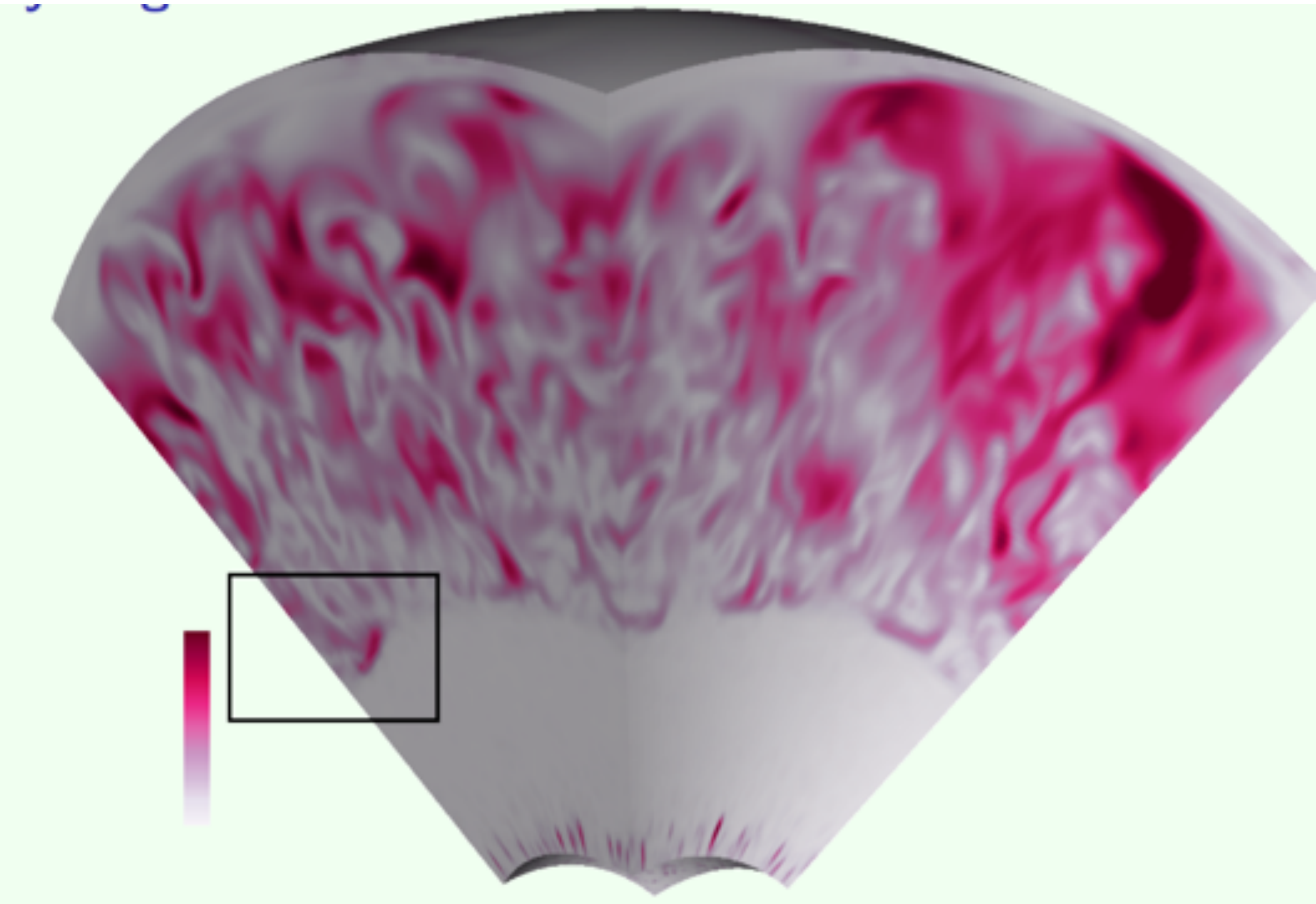


The importance of running over many many convective turnover timescales

☛ Get enough data for a relevant statistical analysis

Application to the **overshooting** problem in stars

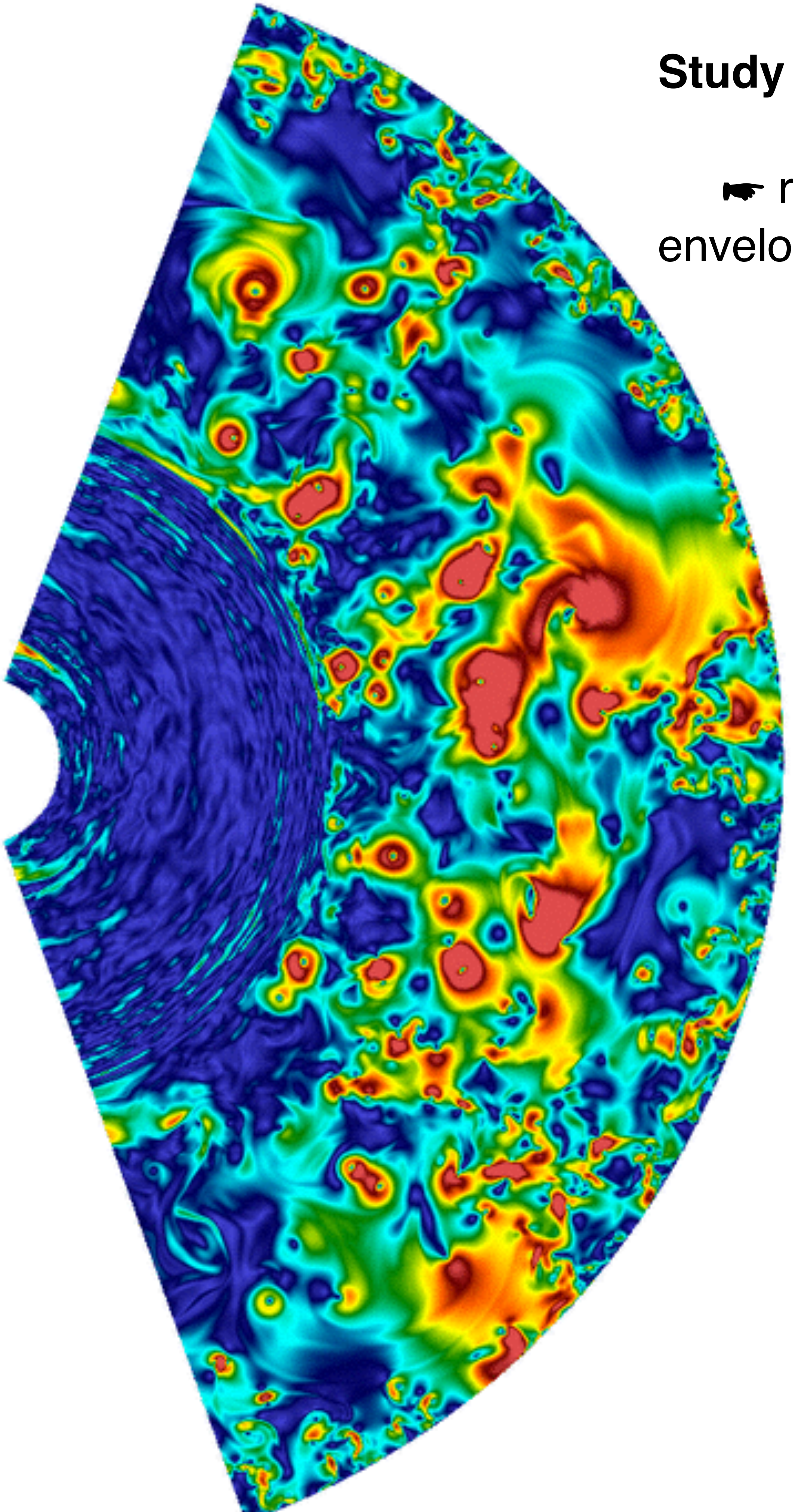
(Long standing problem affecting mixing, transport of angular momentum and magnetic field. Great constraints from asteroseismology)



Velocity magnitude during steady-state convection, in a 128^3 wedge of our sun at $\sim$ 1 Myr, simulated with a the three-dimensional, time-implicit, compressible LES, stellar evolution code MUSIC (ERC Advanced grant, University of Exeter).

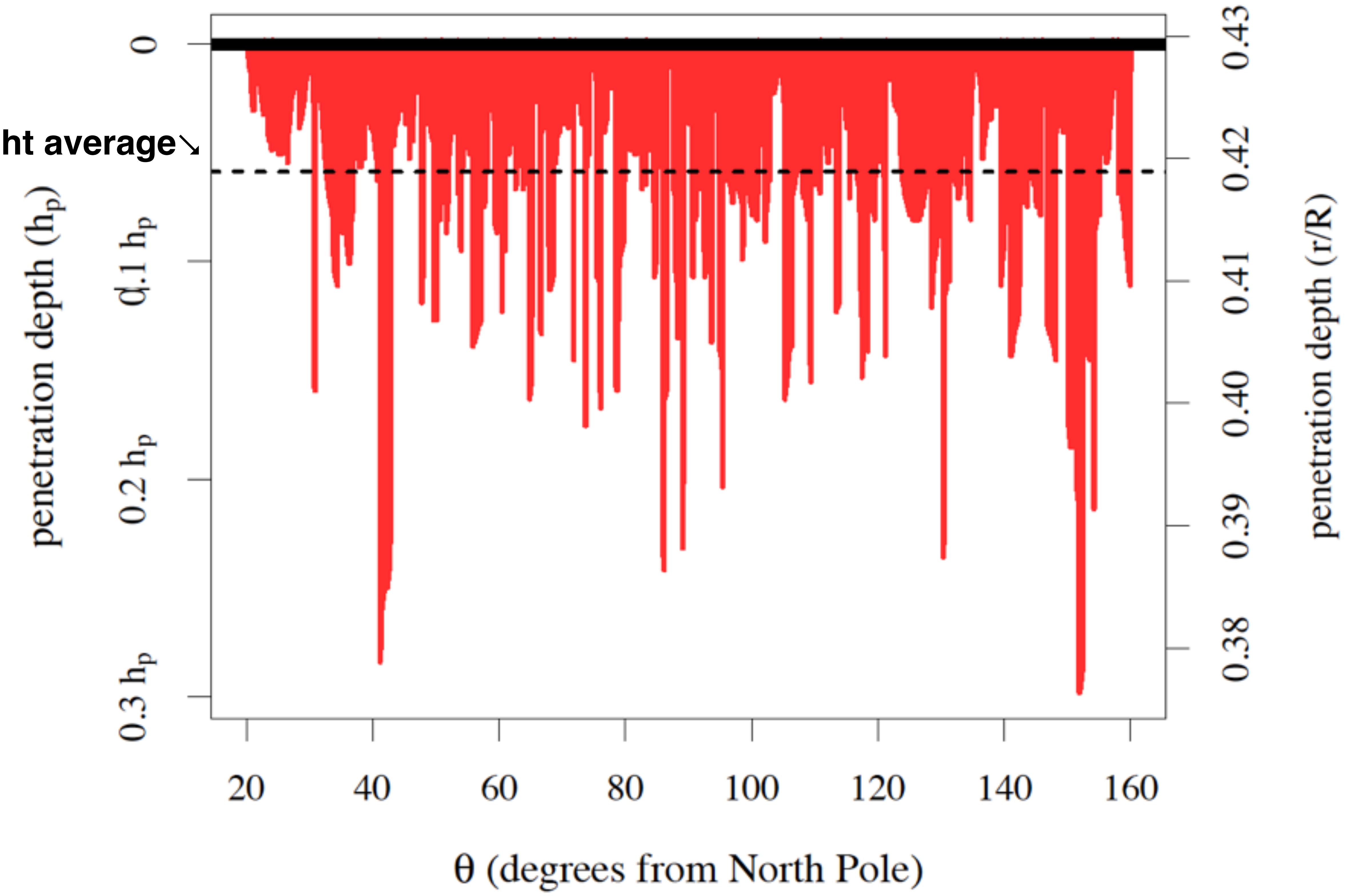
Study of envelope overshooting (Pratt et al. 2016, 2017)

☛ range of 2D/3D simulations of a star with a convective envelope and a radiative core (Pre-main sequence star)



Velocity magnitude : 2D high res 2432x2048

Typical shape of the penetration depths (at a given time): extent of downflows beyond the convective boundary varies with colatitude θ

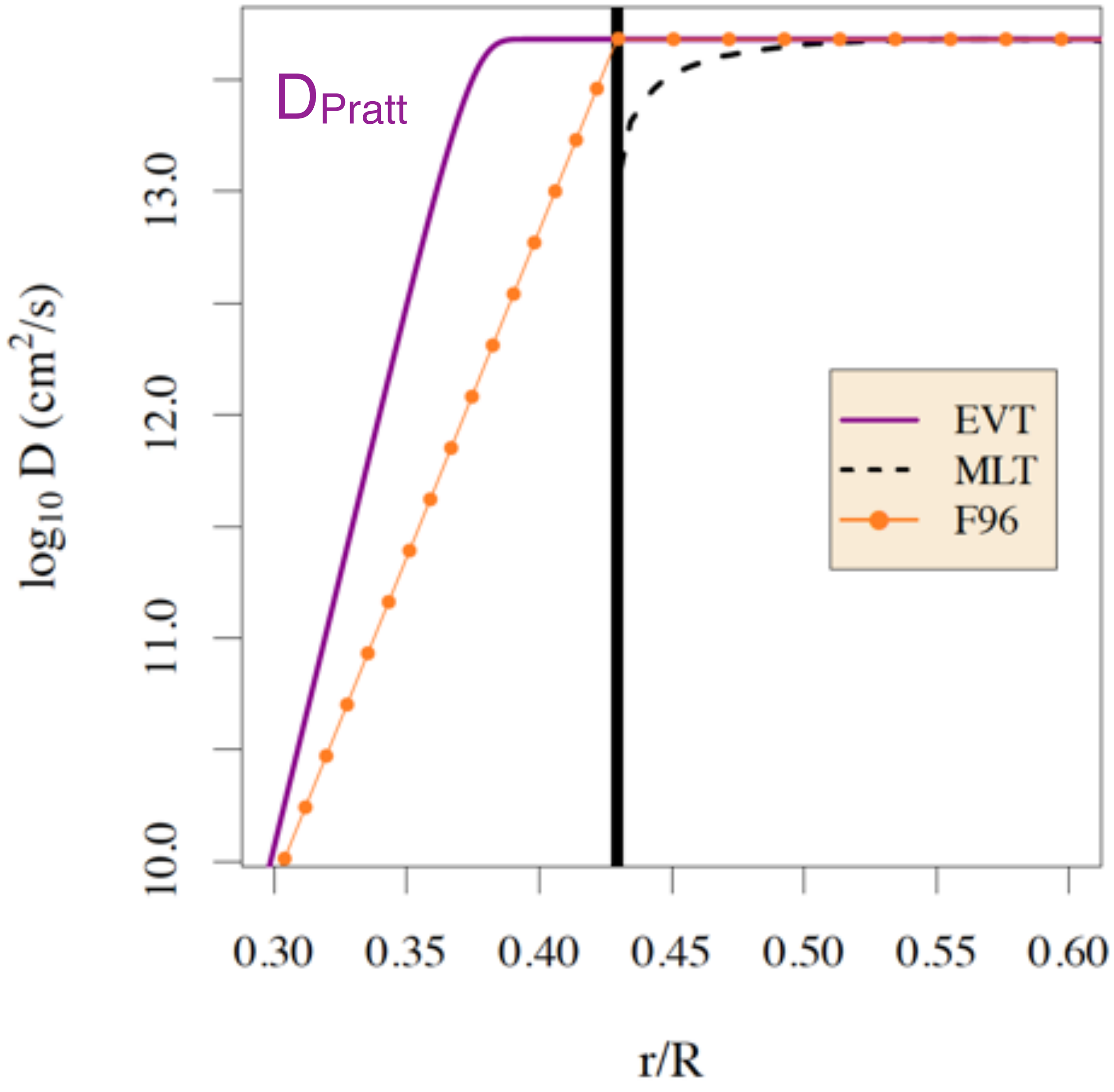


straight average↘

☛ **Straight average miss the larger penetration events**

To perform a statistical analysis of plumes: **several hundreds** (up to ~ 500) of convective turnover timescales $\tau_{\rm conv} \sim 3\ 10^6$ s ($\tau_{\rm dyn} \sim 4$ hr)

Better description of the statistical complexity of the data: **Extreme Value Theory**
Determine the probability of events that are more extreme than any previously observed (used in Earth science, traffic prediction, unusually large flooding event, finance…)



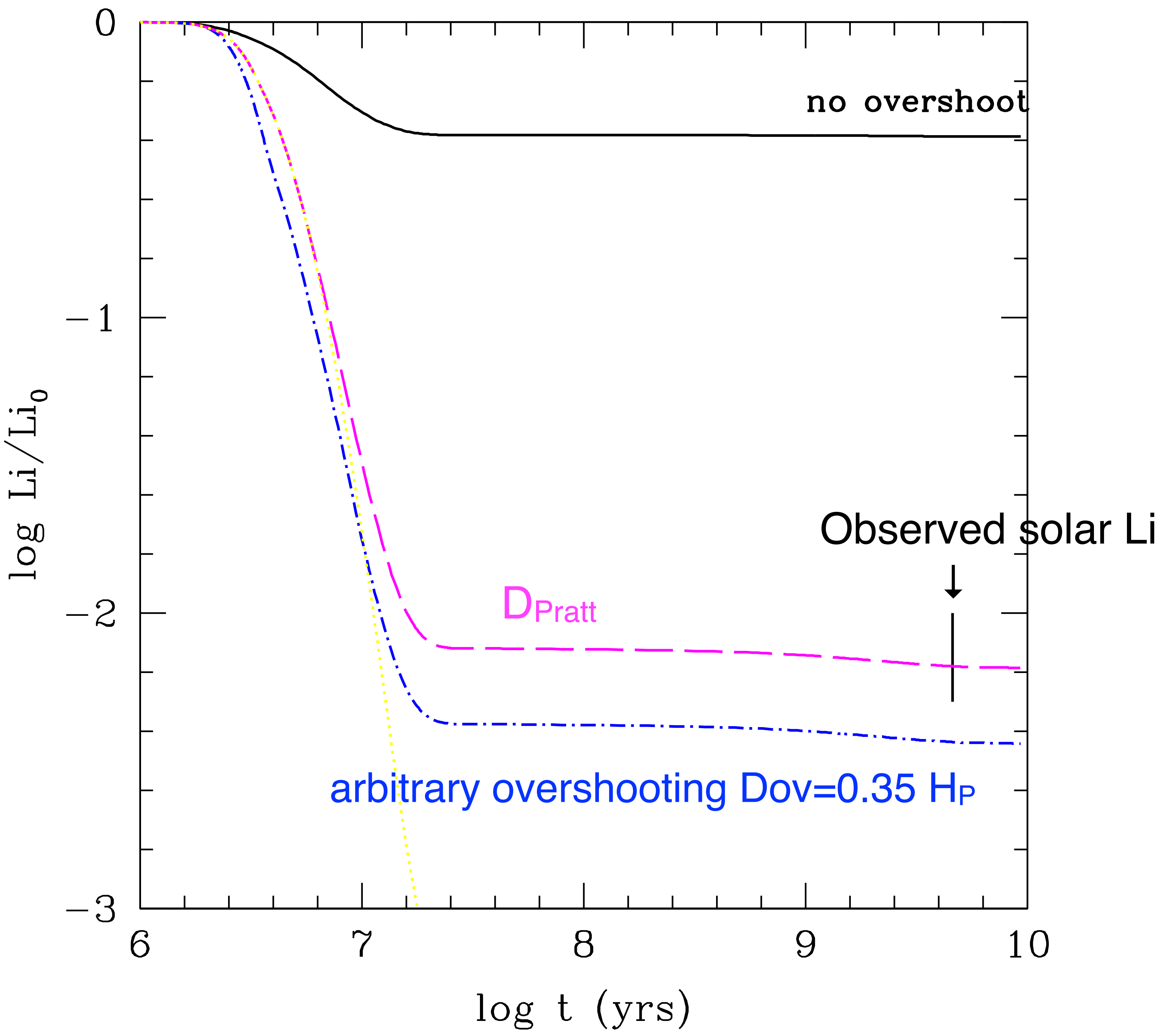
☛ Distribution of maximal penetration depths, linked to extreme events in the tail of the distribution → **contribute to mixing**

☛ Derivation of a diffusion coefficient D(r) characterising the mixing driven by penetrative plumes

$$D_{\rm Pratt}(r) = D_0[1 - exp(-exp(f(r)))]$$

(Pratt et al. A&A, 2017, submitted)

Application to the depletion of Li in the Sun convective envelope



(Baraffe, Pratt et al. A&A, in prep)

Summary of the advantages of MUSIC approach for stellar physics:

- **Spherical coordinates** natural for stellar interiors
(no box-in-a-star approach)

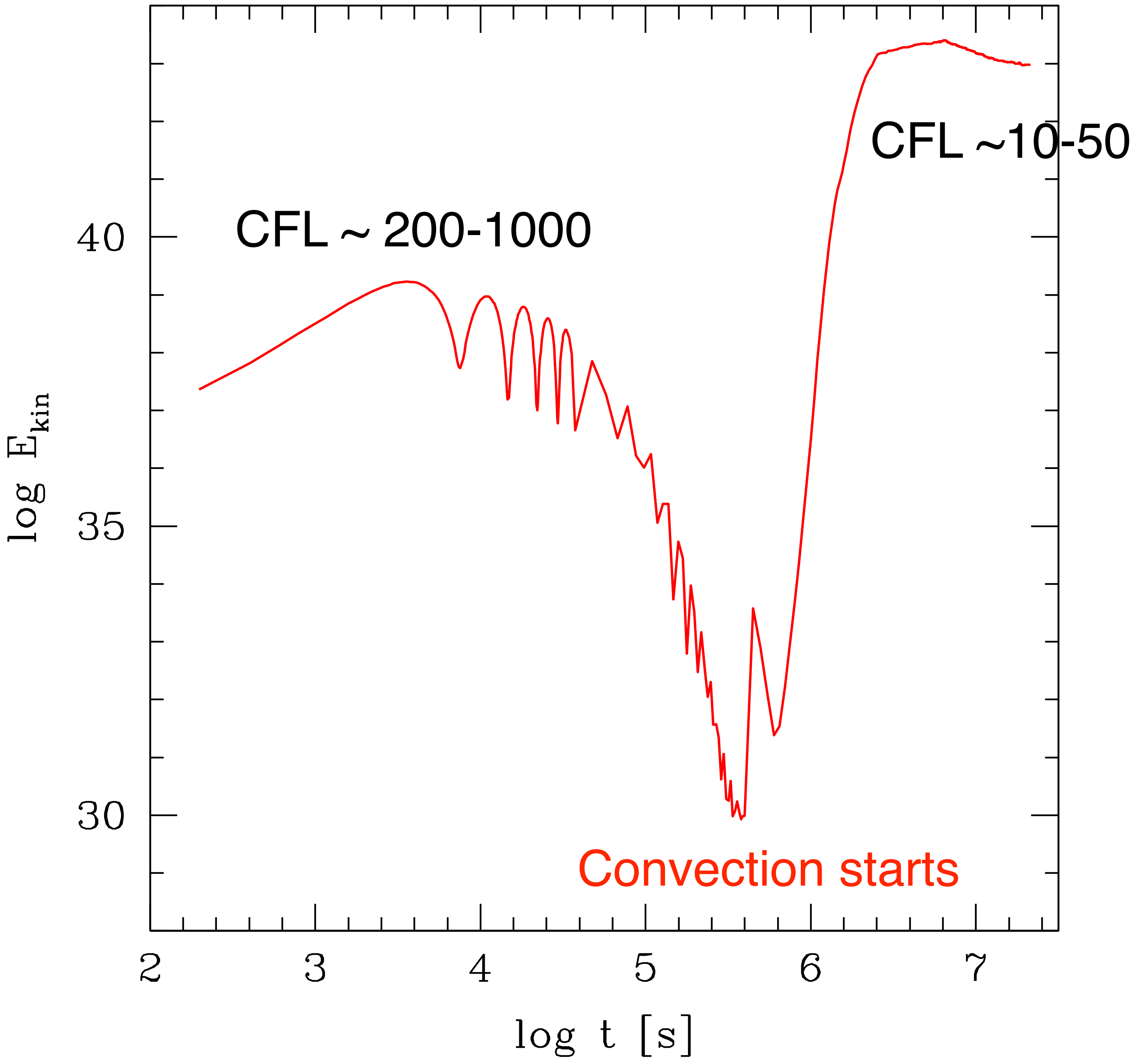
- **Realistic input physics** (EOS/opacities) and direct link with 1D stellar evolution initial models (spherical symmetry)

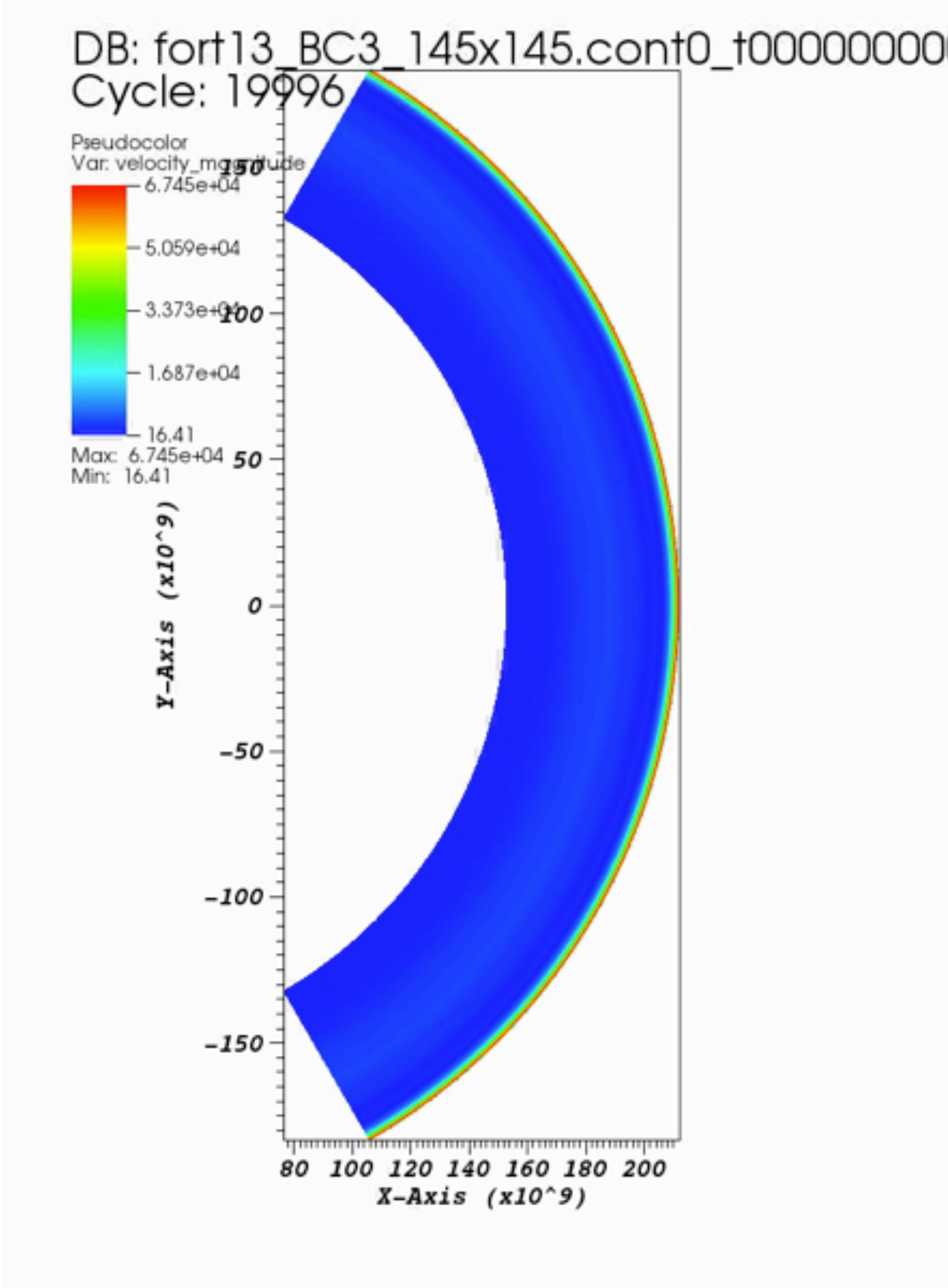
- Solve **fully compressible** hydrodynamical equations
(background can evolve and depart from adiabatic, in opposite to anelastic proxy)

- **Time implicit solver**
  —> large timesteps (CFL ~ 10-1000)
  —> Cover a range of Mach numbers: from centre (M << 1) to surface M ~ 1
  (no need for truncating the surface e.g anelastic codes)

—> speed up the relaxation phase starting from 1D initial model
(large time steps + stability)
*(**key** to explore a range of parameters)*

Relaxation phase from a 1D initial stellar model





Disadvantages

(1) Choice of time step is crucial (accuracy, convergence, performance) and problem/star dependent

(2) Physics based preconditionner adapted to present physics (no MHD)

(3) 3D simulations still too CPU time consuming

Next steps

- **Improve the solver (multi-grid methods?)**
- **(Adaptive) mesh refinement (surface) ?**
- **Study rotation effects (already included in MUSIC)**
- **MHD (long term)**

Main conclusion: despite many skepticism/challenges, the concept of a **time implicit fully compressible hydrodynamic code** works (MUSIC). It begins to bear fruits and to provide improvement of phenomenological approaches used in 1D stellar evolution codes.

☛ It has formidable potentials