

GRAND CHALLENGES IN COMPUTATIONAL ASTROPHYSICS: DUST, MHD AND RADIATION

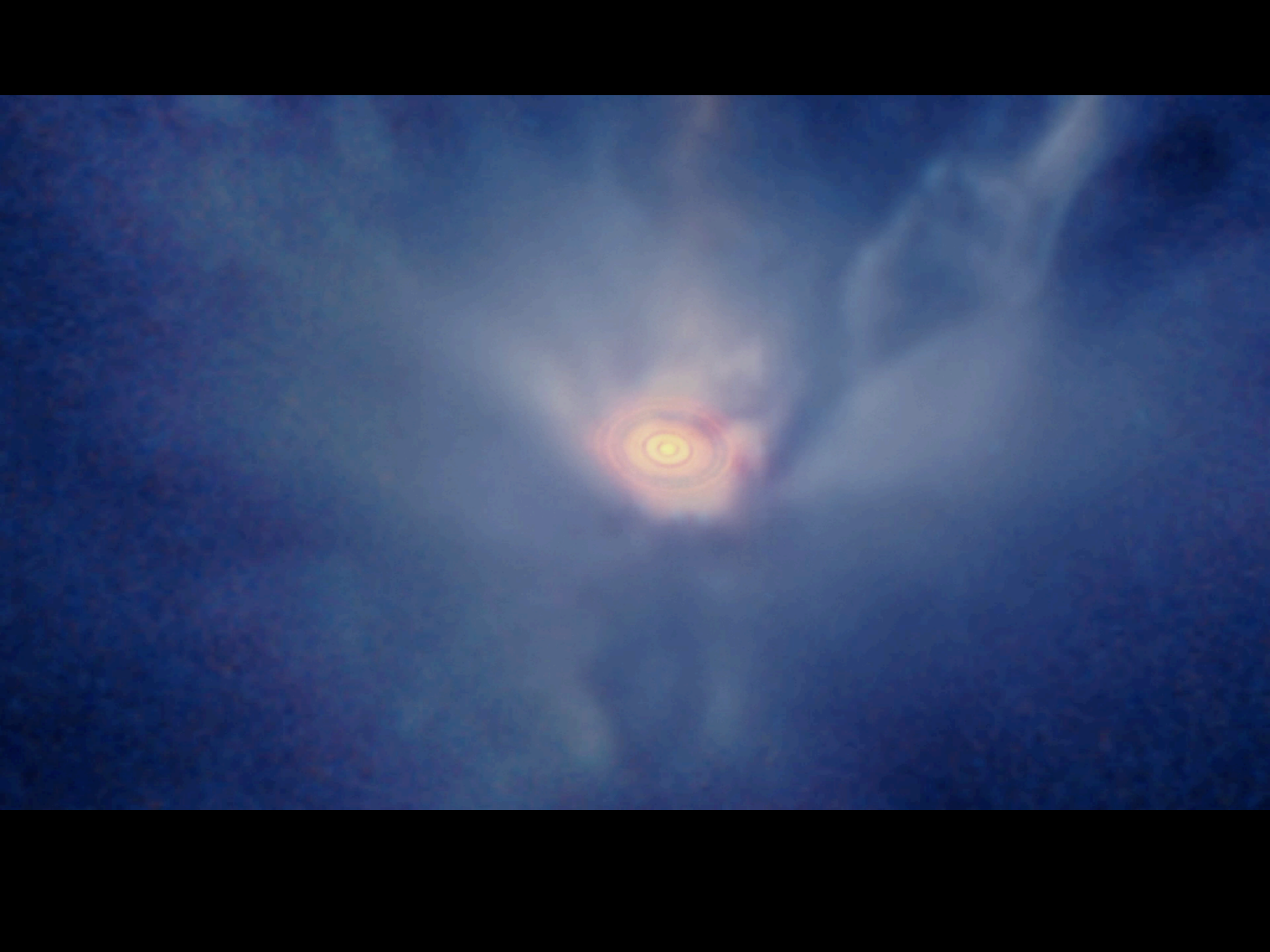


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*Carving through the codes: Challenges in Computational Astrophysics
12-17th February 2017, Davos, Switzerland*



DUST

TWO FLUID

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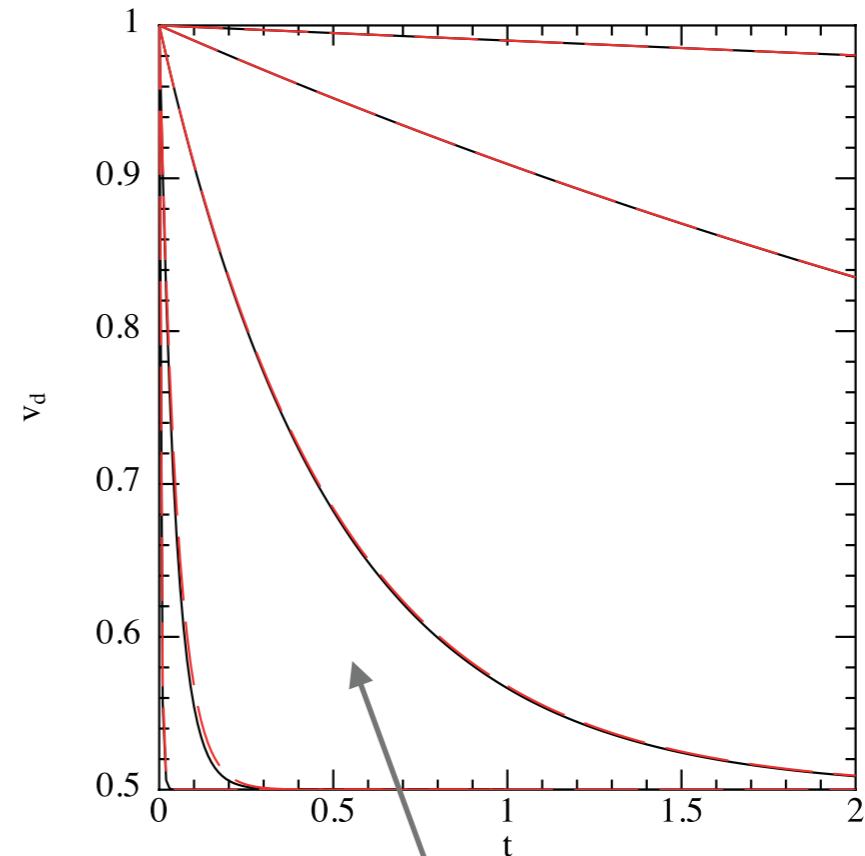
$$\frac{\partial \rho_g}{\partial t} + \nabla \cdot (\rho_g \mathbf{v}_g) = 0,$$

$$\frac{\partial \rho_d}{\partial t} + \nabla \cdot (\rho_d \mathbf{v}_d) = 0,$$

$$\frac{\partial \mathbf{v}_g}{\partial t} + (\mathbf{v}_g \cdot \nabla) \mathbf{v}_g = -\frac{\nabla P_g}{\rho_g} + K(\mathbf{v}_d - \mathbf{v}_g) + \mathbf{f},$$

$$\frac{\partial \mathbf{v}_d}{\partial t} + (\mathbf{v}_d \cdot \nabla) \mathbf{v}_d = -K(\mathbf{v}_d - \mathbf{v}_g) + \mathbf{f},$$

Timestep constraint: $\Delta t < t_s$



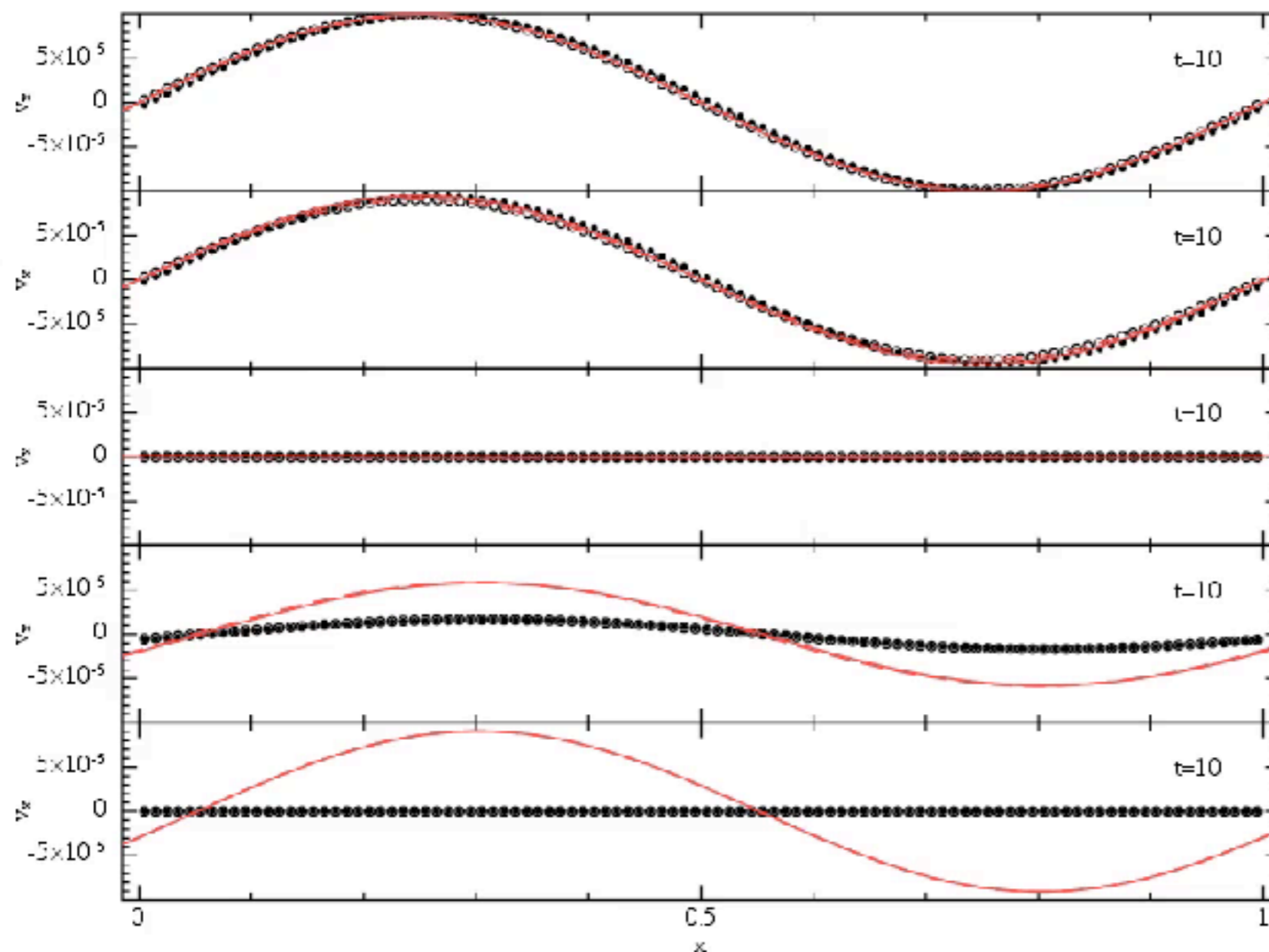
Characteristic timescale t_s

DUSTYWAVE

Maddison (1993), Laibe & Price (2011)

$$(\omega^2 - c_s^2 k^2) - \frac{i}{\omega t_s} (\omega - \tilde{c}_s^2 k^2) = 0$$

1) OVERDAMPING PROBLEM



*No drag = no damping
numerical = exact*

*Intermediate drag =
strong damping in both
numerical and exact*

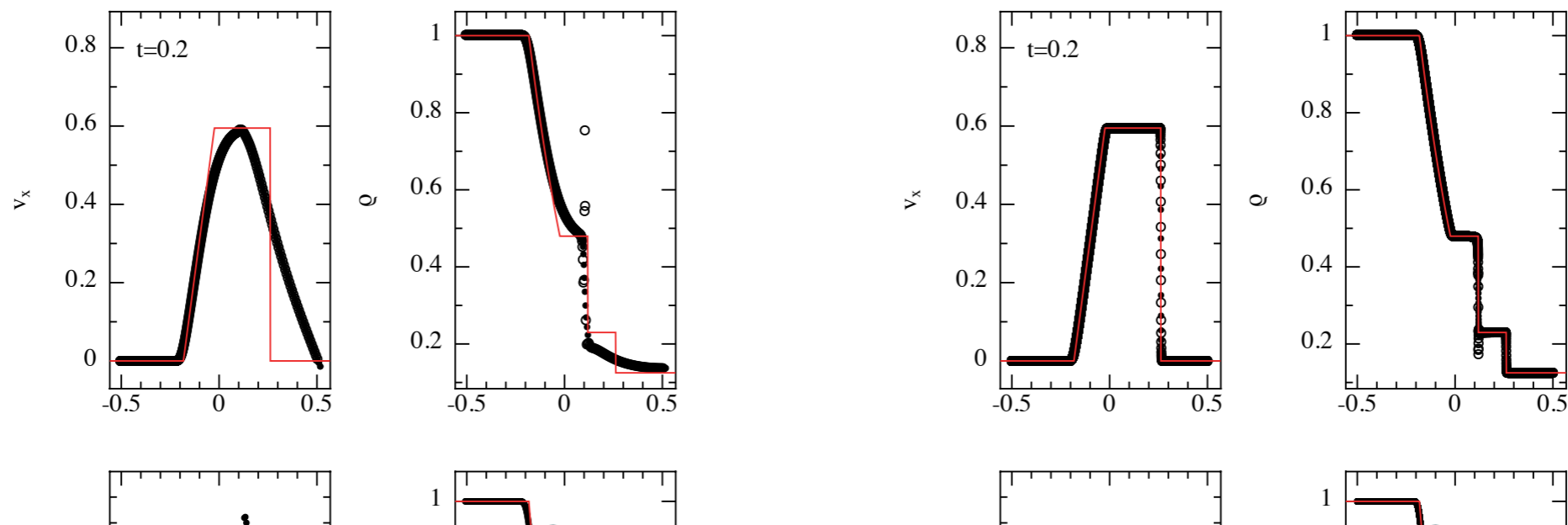
*High drag =
no damping*

*but numerical solution
strongly damped*

*Red = analytic solution for dust/gas waves derived by
Laibe & Price (2011) MNRAS 418, 1491*

OVERDAMPING PROBLEM

Must resolve stopping length
 $L \sim cs \, ts$



“The hybrid scheme ... is second order only in the non-stiff regime ... the drop in accuracy ... is most likely due to the difficulty of coupling the gas and the dust fully self-consistently in the stiff regimes.” (Miniati 2010, J Comp Phys)

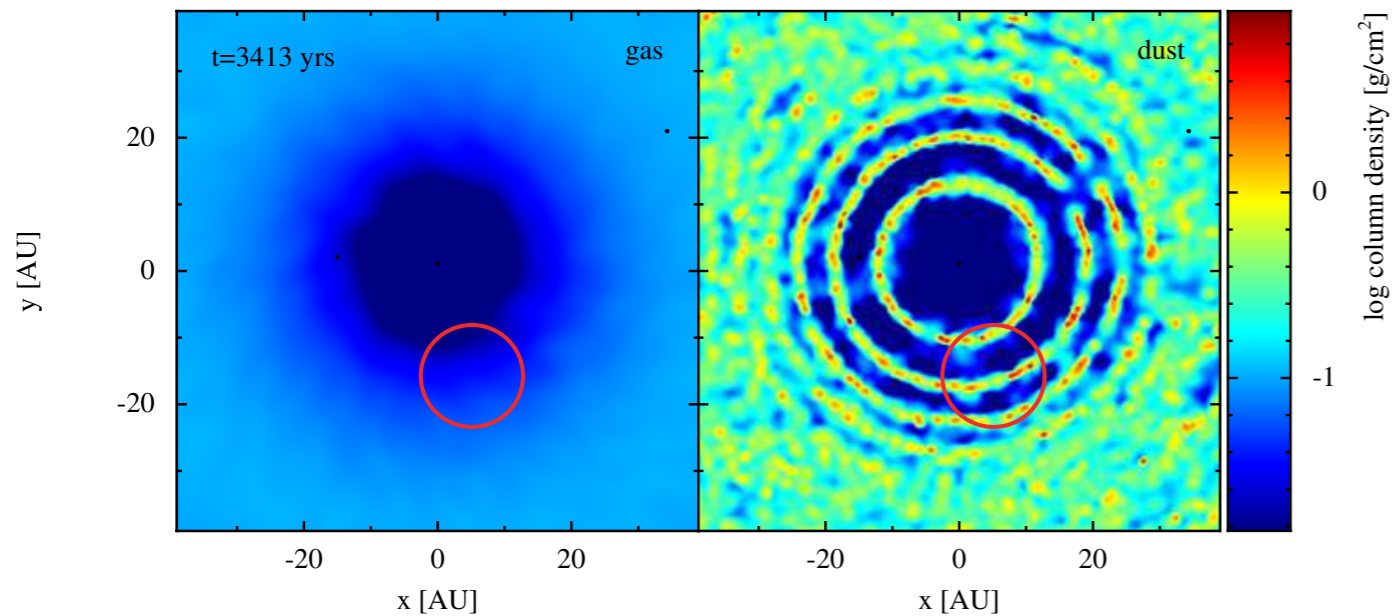


sensible resolution

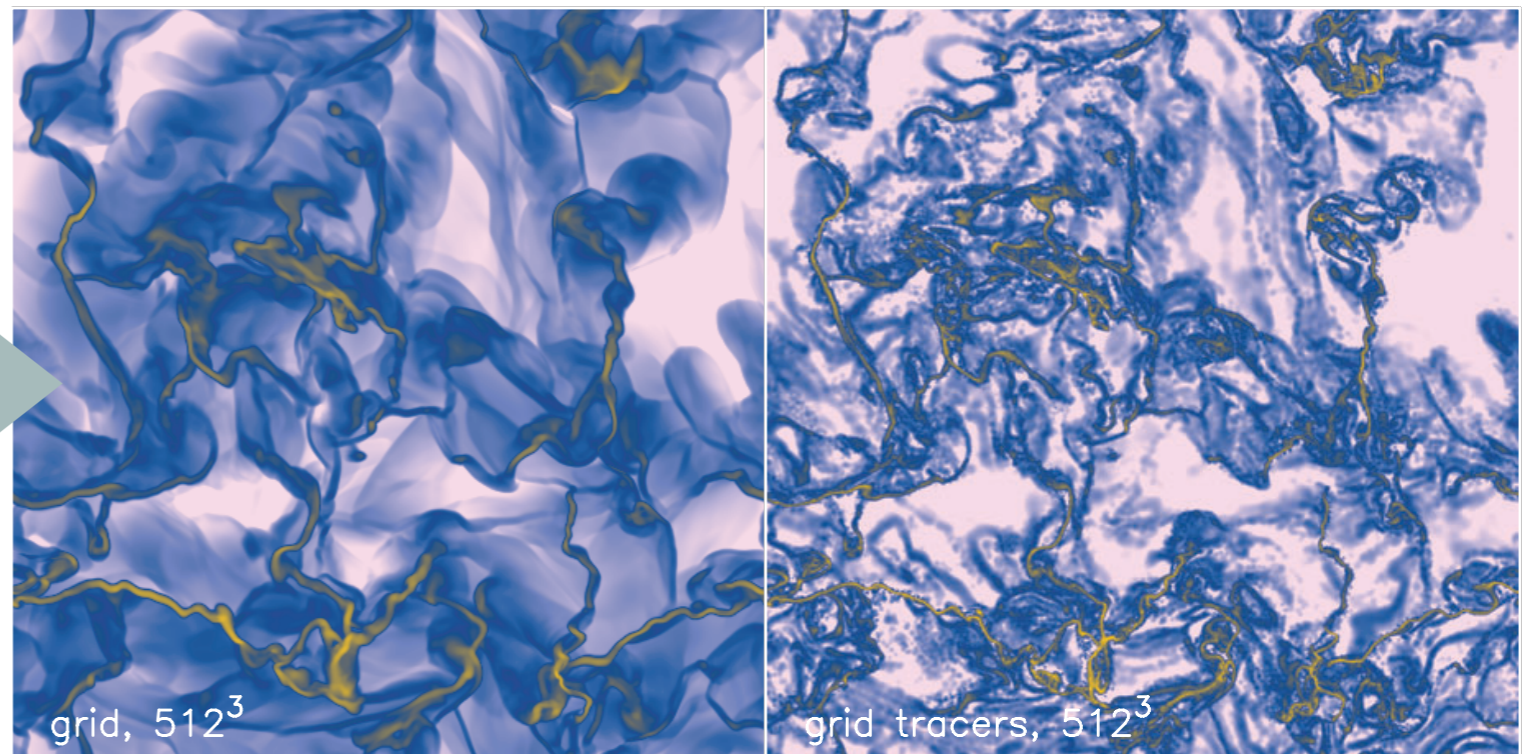
ludicrous resolution

II. DUST TRAPPING PROBLEM

- Dust particles get 'stuck' below gas resolution length



Price & Federrath (2010):
same issue for tracer/
dust particles on grids



DUST-GAS: ONE FLUID

One mixture with a differential velocity

$$\rho = \rho_d + \rho_g$$

$$\epsilon = \rho_d / \rho$$

$$\mathbf{v} = \frac{\rho_d \mathbf{v}_d + \rho_g \mathbf{v}_g}{\rho}$$

$$\Delta \mathbf{v} = \mathbf{v}_d - \mathbf{v}_g$$

$$\frac{d\rho_g}{dt} + \nabla \rho (\nabla_g \mathbf{v}_g) = 0,$$

$$\frac{d\epsilon}{dt} + \nabla \cdot \left(\frac{1}{\rho} (\rho_d \mathbf{v}_d) (1 - \epsilon) \right) = 0,$$

$$\frac{d\mathbf{v}_g}{dt} = - \left(\mathbf{v}_g \cdot \nabla \right) \mathbf{v}_g - \frac{1}{\rho} \nabla \cdot \left[\epsilon \left(1 - \frac{\nabla P_g}{\rho_g} \right) + \frac{K}{\rho_g} \Delta \mathbf{v} \right] + \mathbf{f}_g +$$

$$\frac{d\Delta \mathbf{v}}{dt} = - \left(\mathbf{v}_d \cdot \nabla \right) \mathbf{v}_d - \frac{\nabla P}{\rho_g} - \left(\frac{K}{\rho_d} (\nabla_d) \mathbf{v}_d + \frac{1}{2} \nabla \cdot [(2\epsilon - 1) \Delta \mathbf{v}^2] \right).$$

ONE FLUID FOR SMALL GRAINS

Laibe & Price (2014), Price & Laibe (2015)

.....

$$\frac{d\rho}{dt} = -\rho(\nabla \cdot \mathbf{v})$$

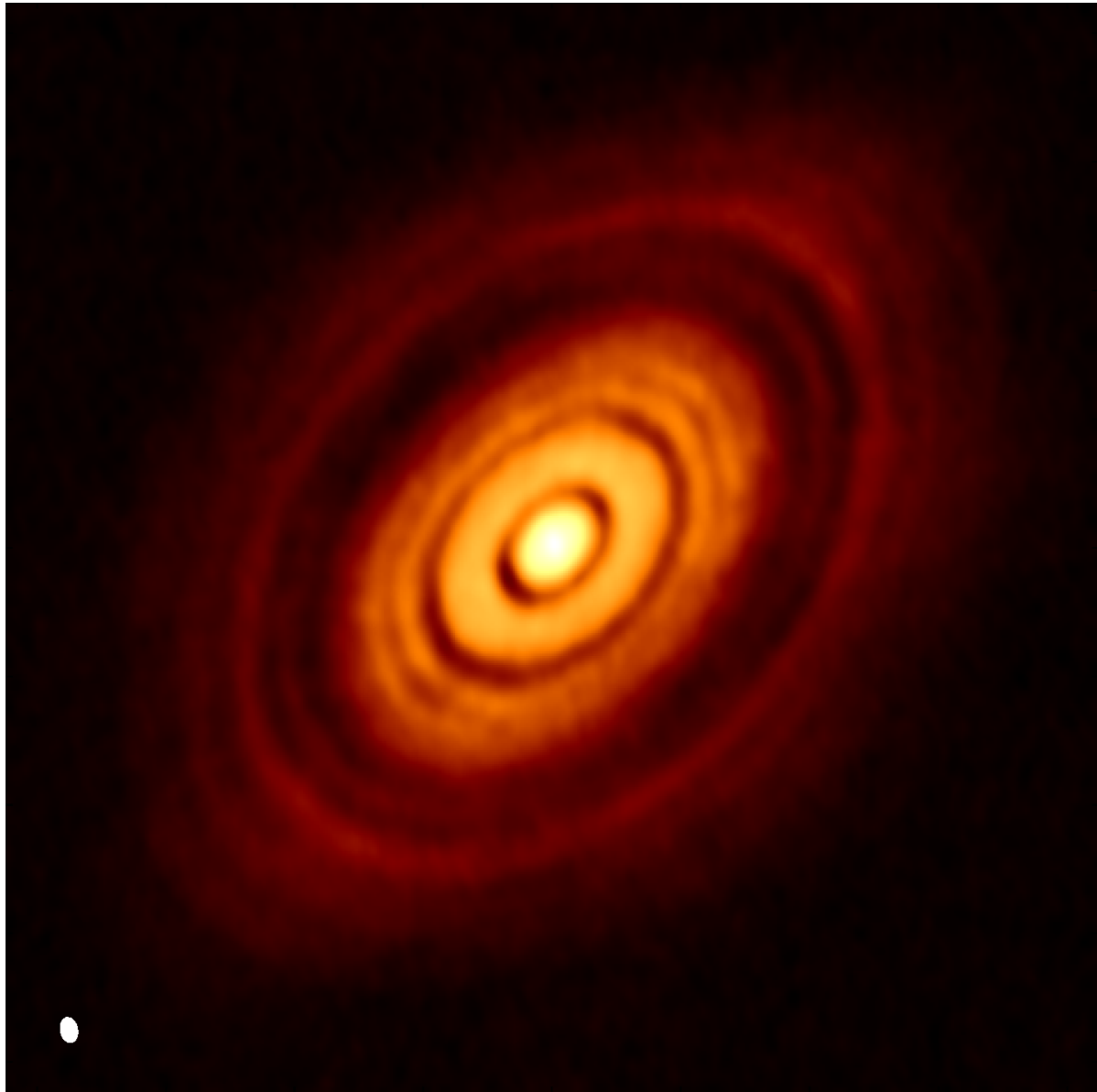
$$\frac{d\epsilon}{dt} = -\frac{1}{\rho} \nabla \cdot [\epsilon(1 - \epsilon) \nabla P \rho \Delta \mathbf{v}]$$

$$\frac{d\mathbf{v}}{dt} = -\frac{\nabla P}{\rho} - \frac{1}{\rho} \nabla \cdot [\epsilon(1 - \epsilon) \Delta \mathbf{v} \Delta \mathbf{v}]$$

$$\frac{d\Delta \mathbf{v}}{dt} = -\frac{\Delta \mathbf{v}}{t_s} + \frac{\nabla P}{\rho_g} - (\Delta \mathbf{v} \cdot \nabla) \mathbf{v} + \frac{1}{2} \nabla [(2\epsilon - 1) \Delta \mathbf{v}^2]$$

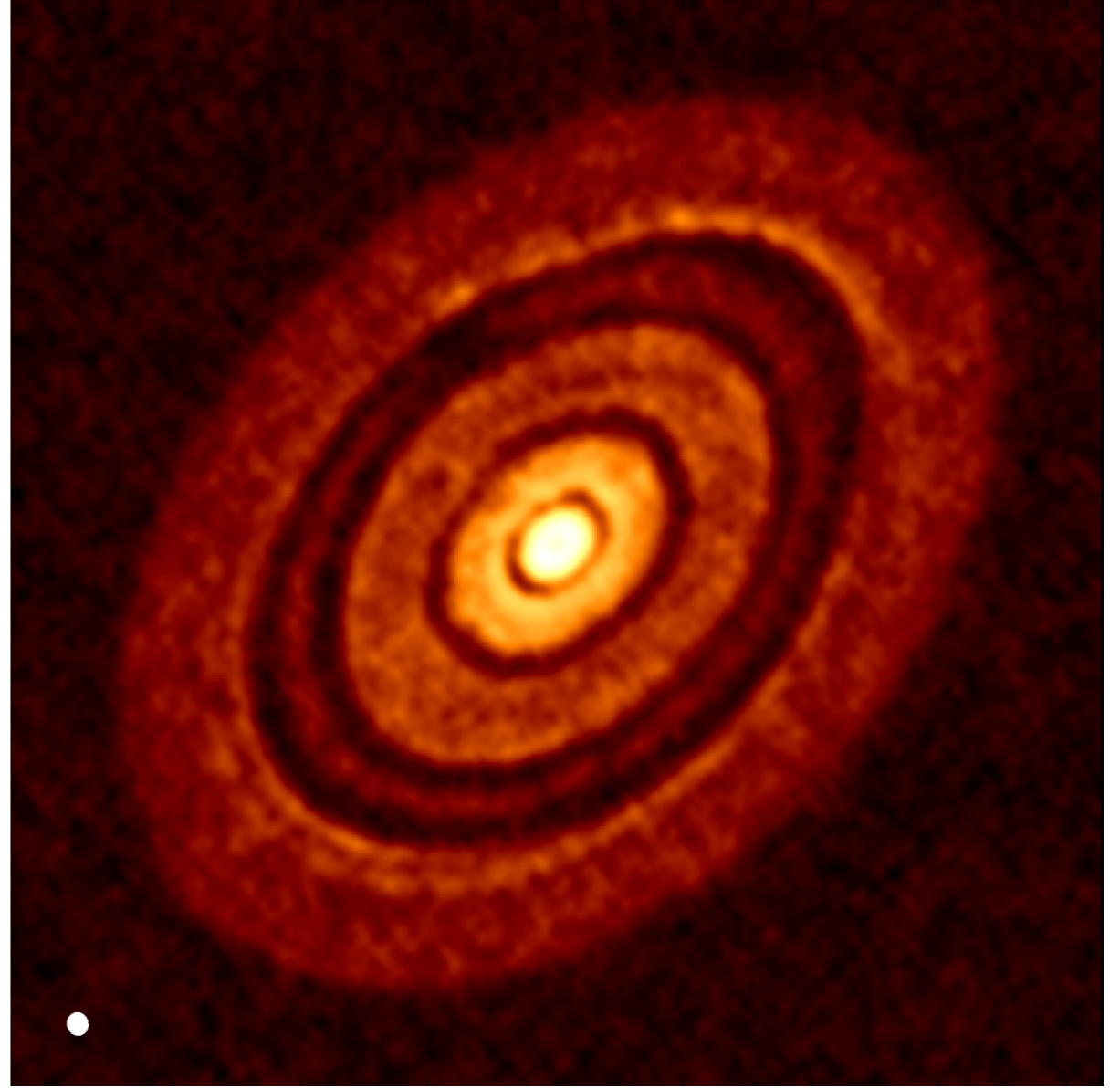
EXPLICIT when stopping time is short

APPLICATION TO HL TAU



Observed image

ALMA partnership et al. (2015)



Our simulation

*Dipierro, Price, Laibe, Hirsh,
Cerioli & Lodato (2015)*

EXAMPLE II: “HORSESHOES” IN TRANSITION DISCS *Ragusa+2017*

4 *Ragusa et al.*

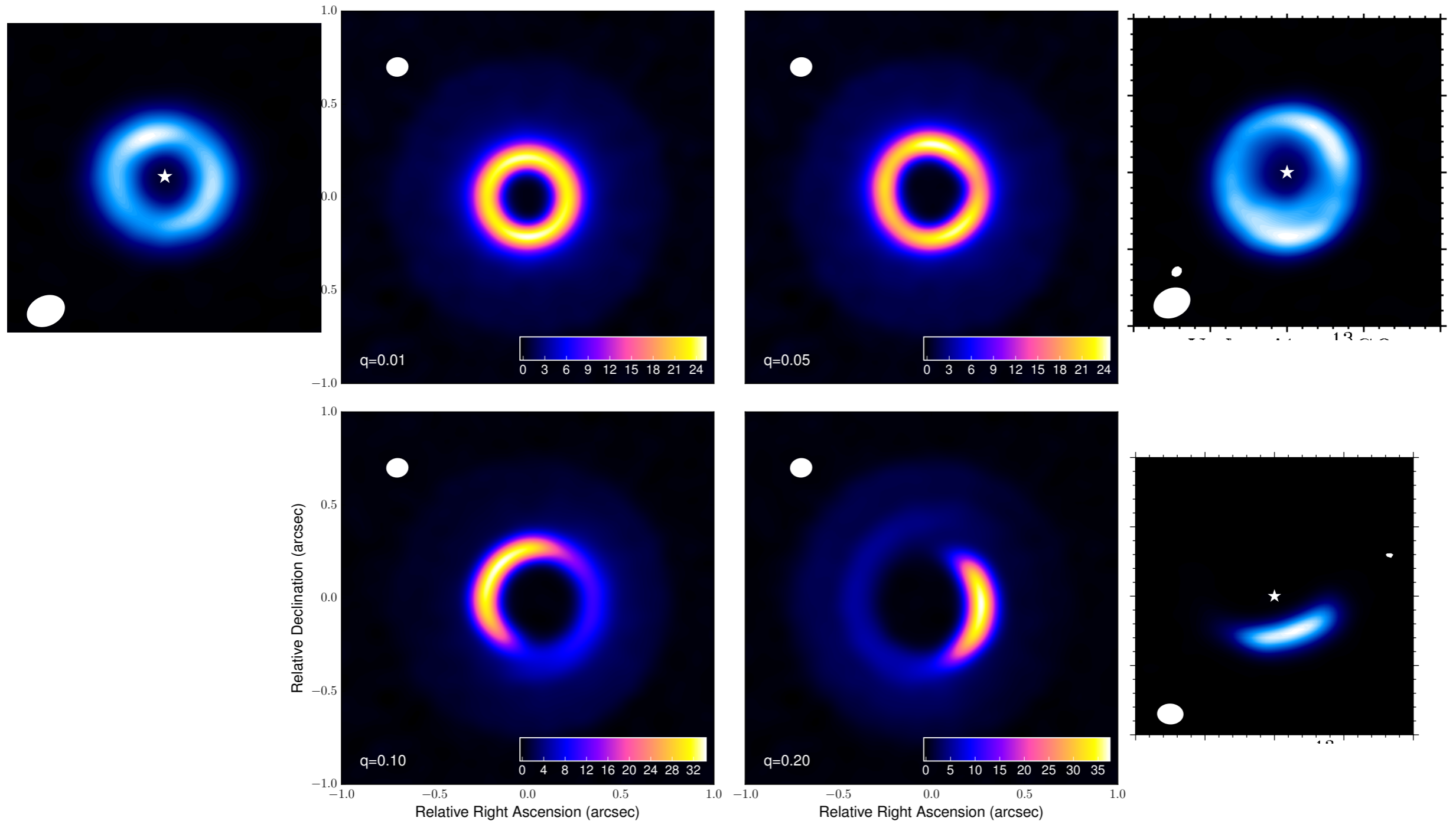


Figure 2. Comparison of ALMA simulated observations at 345 GHz of disc models with a mass ratio $q = 0.01$ (upper left), $q = 0.05$ (upper right), $q = 0.1$ (bottom left) and $q = 0.2$ (bottom right). Intensities are in mJy beam^{-1} . The white colour in the filled ellipse in the upper left corner indicates the size of the half-power contour of the synthesized beam: $0.12 \times 0.1 \text{ arcsec}$ ($\sim 16 \times 13 \text{ au}$ at 130 pc.).

MHD

SMOOTHED PARTICLE MAGNETOHYDRODYNAMICS

see review by Price (2012)
J. Comp. Phys. 231, 759

$$L = \int \left(\frac{1}{2} \rho v^2 - \rho u - \frac{1}{2\mu_0} B^2 \right) dV$$



$$L = \sum_a m_a \left(\frac{1}{2} v_a^2 - u_a - \frac{B_a^2}{2\mu_0 \rho_a} \right)$$

Price & Monaghan
(2004a,b, 2005)

Euler-Lagrange equations give discrete form of:

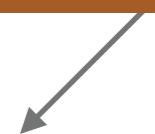
$$\frac{d\rho}{dt} = -\rho(\nabla \cdot \mathbf{v})$$

$$\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho} \nabla \cdot \left[\left(P + \frac{1}{2} \frac{B^2}{\mu_0} \right) \mathbf{I} - \frac{\mathbf{B}\mathbf{B}}{\mu_0} \right] - \frac{\mathbf{B}(\nabla \cdot \mathbf{B})}{\mu_0 \rho}$$

$$\frac{du}{dt} = -\frac{P}{\rho}(\nabla \cdot \mathbf{v})$$

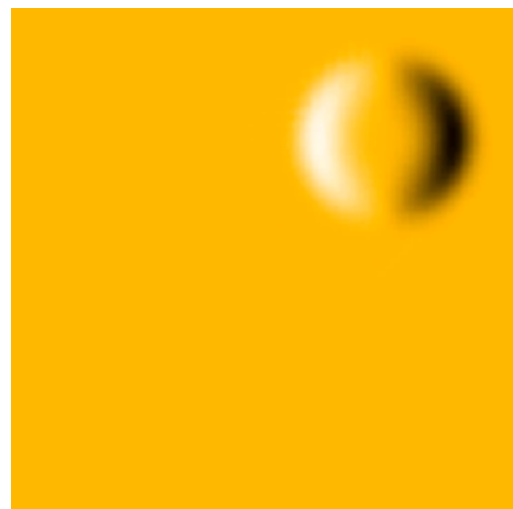
$$\frac{d}{dt} \left(\frac{\mathbf{B}}{\rho} \right) = \left(\frac{\mathbf{B}}{\rho} \cdot \nabla \right) \mathbf{v}$$

Subtract div B
source term for
stability



Dissipationless: Must add dissipation
terms to handle shocks and discontinuities

Need to separately handle $\text{div } \mathbf{B} = 0$



Divergence advection test
from Dedner et al. (2002)

These equations are equivalent to the “8-wave formulation” of Powell et al. 1994

HYPERBOLIC/PARABOLIC DIVERGENCE CLEANING

Dedner et al. (2002)
Price & Monaghan (2005)
Mignone & Tzeferacos (2010)

$$\frac{d\mathbf{B}}{dt} = -\nabla\psi$$

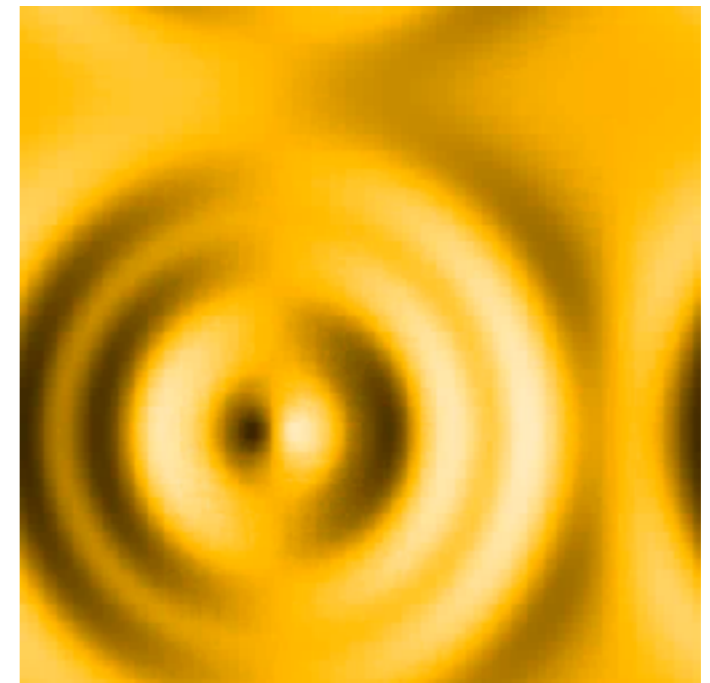
$$\frac{d\psi}{dt} = -c_h^2(\nabla \cdot \mathbf{B}) - \frac{\sigma c_h}{h}\psi$$

Hyperbolic

Parabolic



$$\frac{1}{c_h^2} \frac{\partial^2(\nabla \cdot \mathbf{B})}{\partial t^2} + \nabla^2(\nabla \cdot \mathbf{B}) + \frac{1}{\lambda c_h} \frac{\partial(\nabla \cdot \mathbf{B})}{\partial t} = 0$$



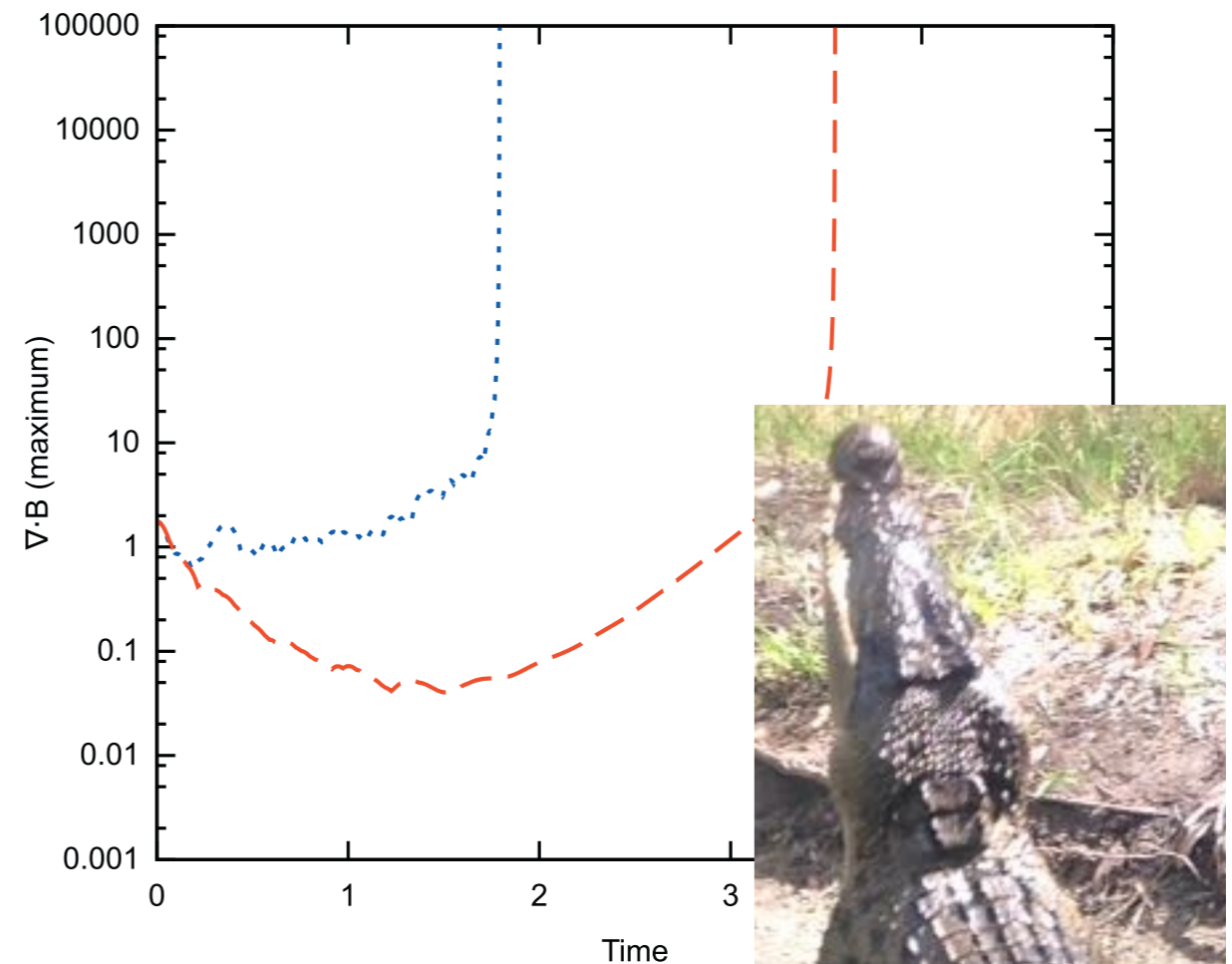
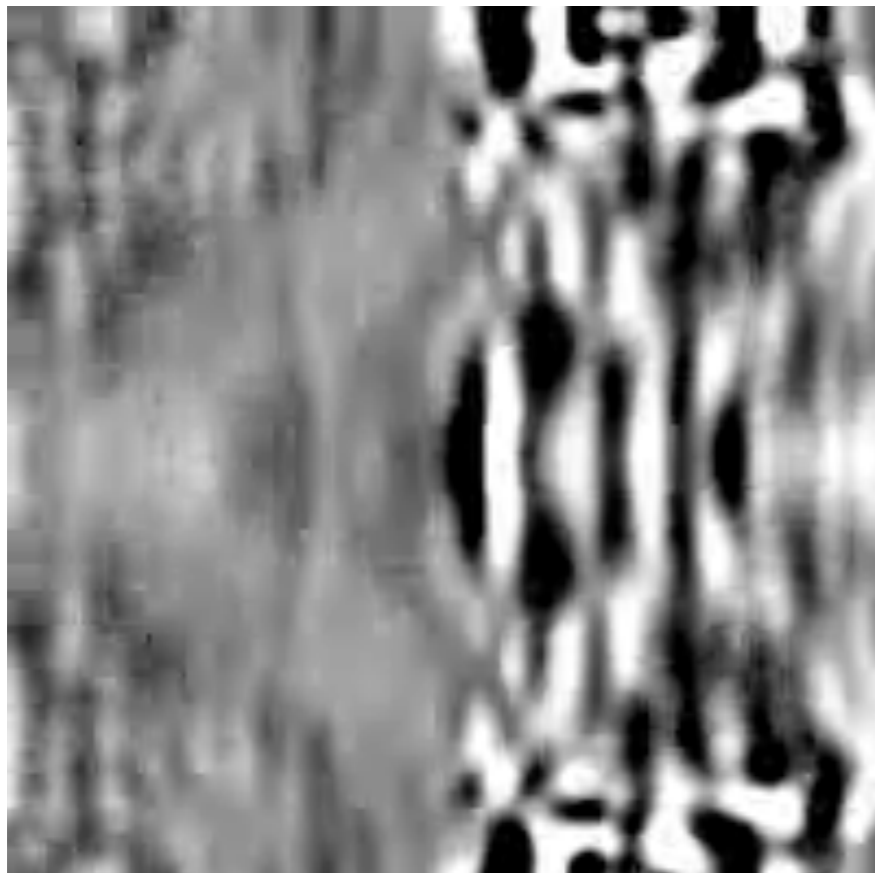
Hyperbolic term only

WHEN CLEANING ATTACKS

.....

7224

T.S. Tricco, D.J. Price/*Journal of Compu*



*Divergence advection test (Dedner et al. 2002)
with 10:1 jump in density*

“CONSTRAINED” HYPERBOLIC/PARABOLIC DIVERGENCE CLEANING

Tricco & Price (2012); Tricco, Price & Bate (2016)

- Define energy associated with cleaning field

$$E = \int \left[\frac{1}{2} \frac{B^2}{\mu_0} + \frac{1}{2} \frac{\psi^2}{\mu_0 c_h^2} \right] dV$$

- Enforce energy conservation in hyperbolic terms

$$\frac{dE}{dt} = \int \left[\frac{\mathbf{B}}{\mu_0} \cdot \left(\frac{d\mathbf{B}}{dt} \right)_{\psi} + \frac{\psi}{\mu_0 c_h^2} \frac{d\psi}{dt} - \frac{\psi^2}{2\mu_0 \rho c_h^2} \frac{d\rho}{dt} - \frac{\psi^2}{\mu_0 c_h^3} \frac{dc_h}{dt} \right] dV = 0$$



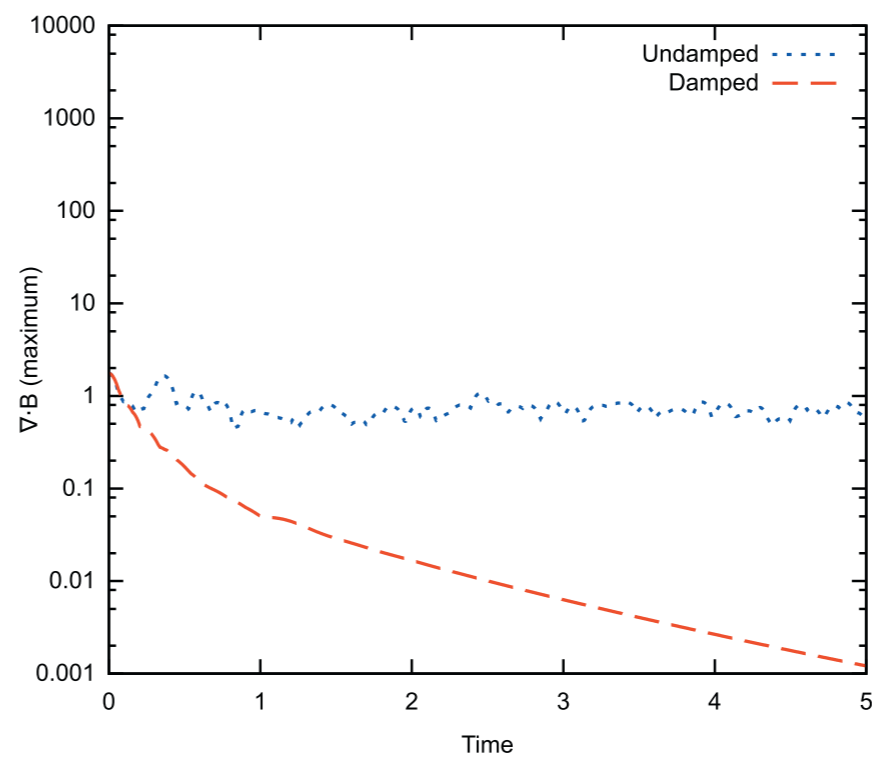
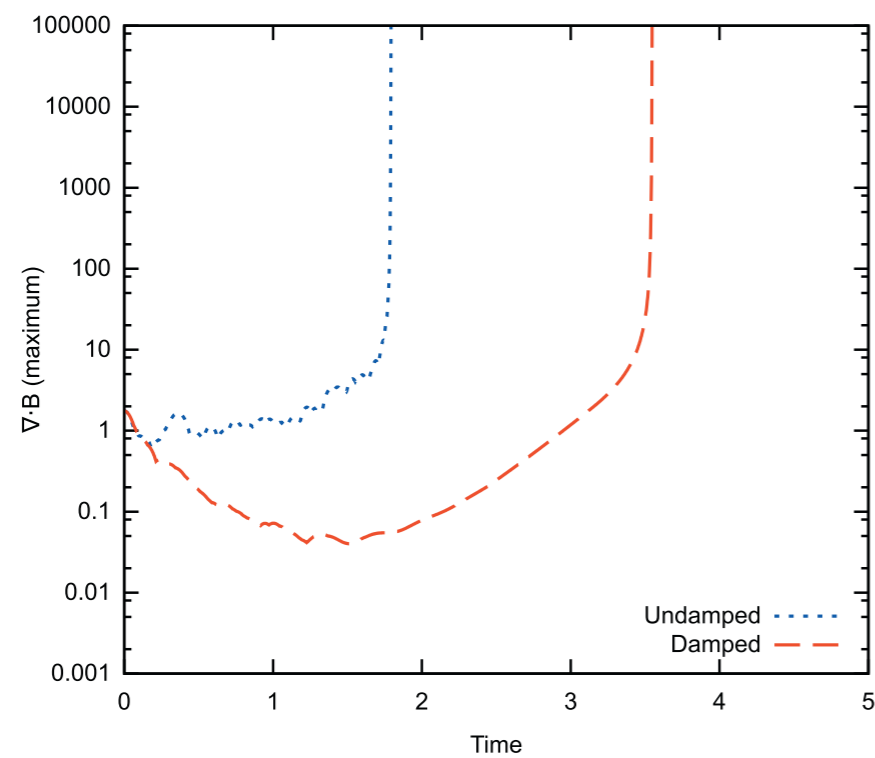
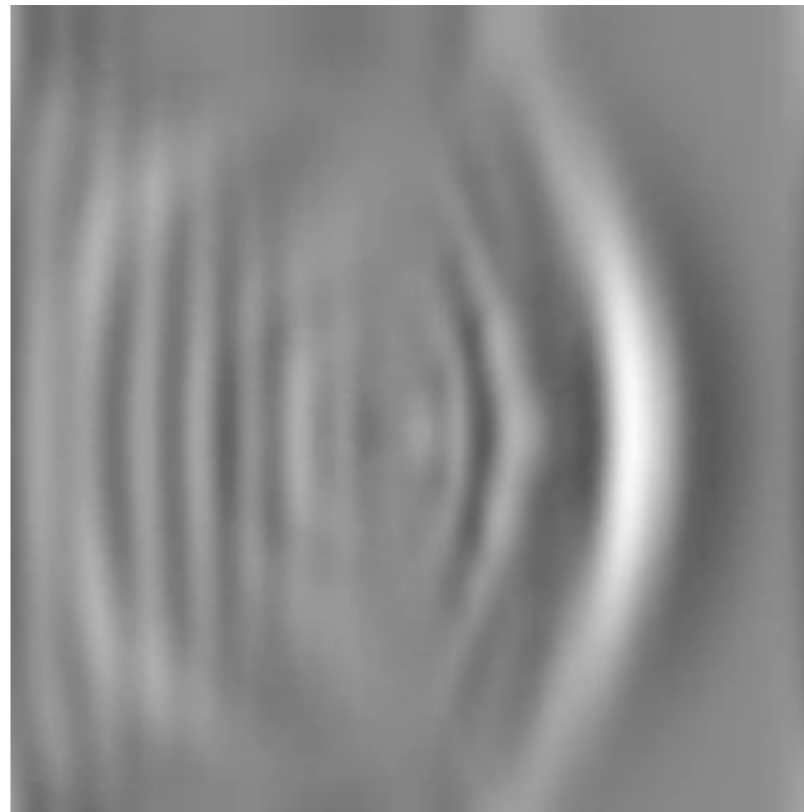
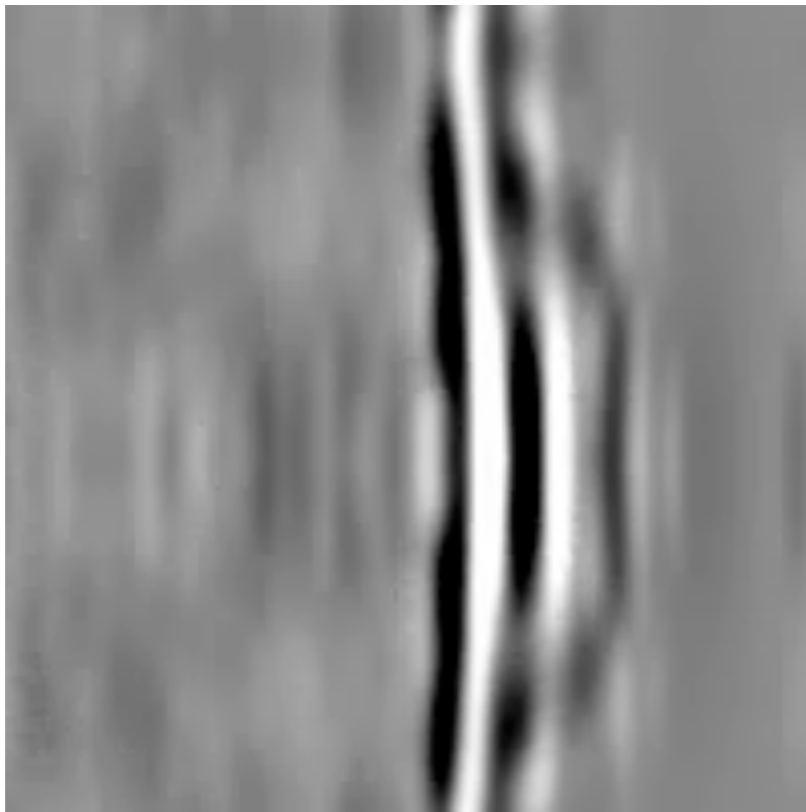
$$\frac{d\mathbf{B}}{dt} = -\nabla \psi$$

$$\frac{d\psi}{dt} = -c_h^2 (\nabla \cdot \mathbf{B}) - \frac{\sigma c_h}{h} \psi - \frac{1}{2} \psi (\nabla \cdot \mathbf{v})$$

Requires particular choice of operators here

- Can enforce exact energy conservation in SPH discretisation

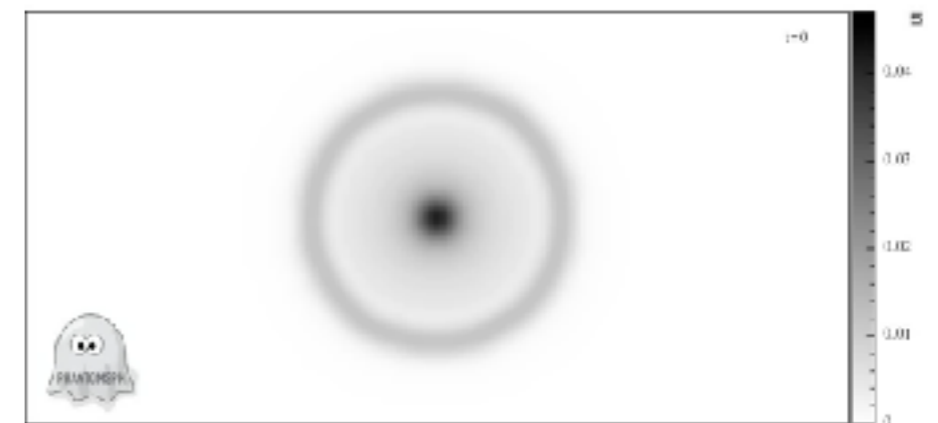
CONSTRAINED HYPERBOLIC/PARABOLIC CLEANING



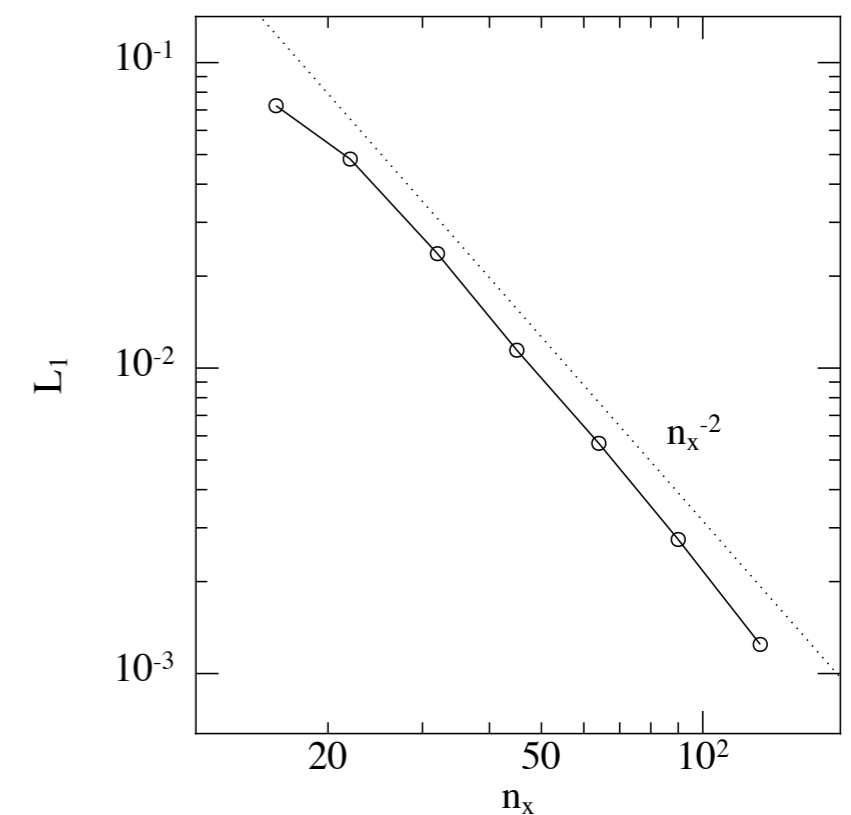
Parabolic term is
negative definite!

PHANTOM SPMHD CODE

Price et al. (2017)



Advection of current loop (Gardiner & Stone 2005, 2008)

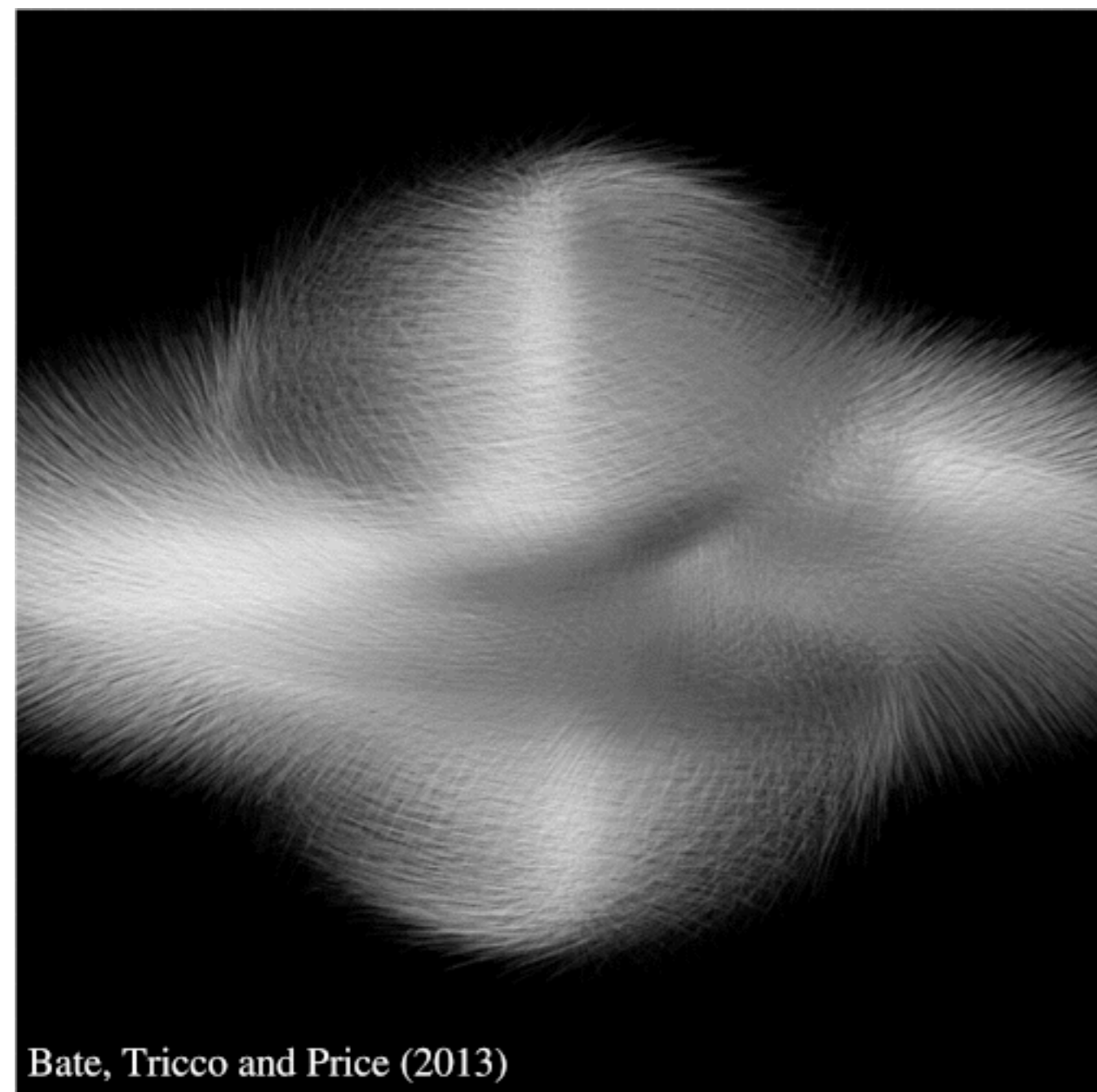


Convergence on circularly polarised Alfvén wave with ALL dissipation turned on

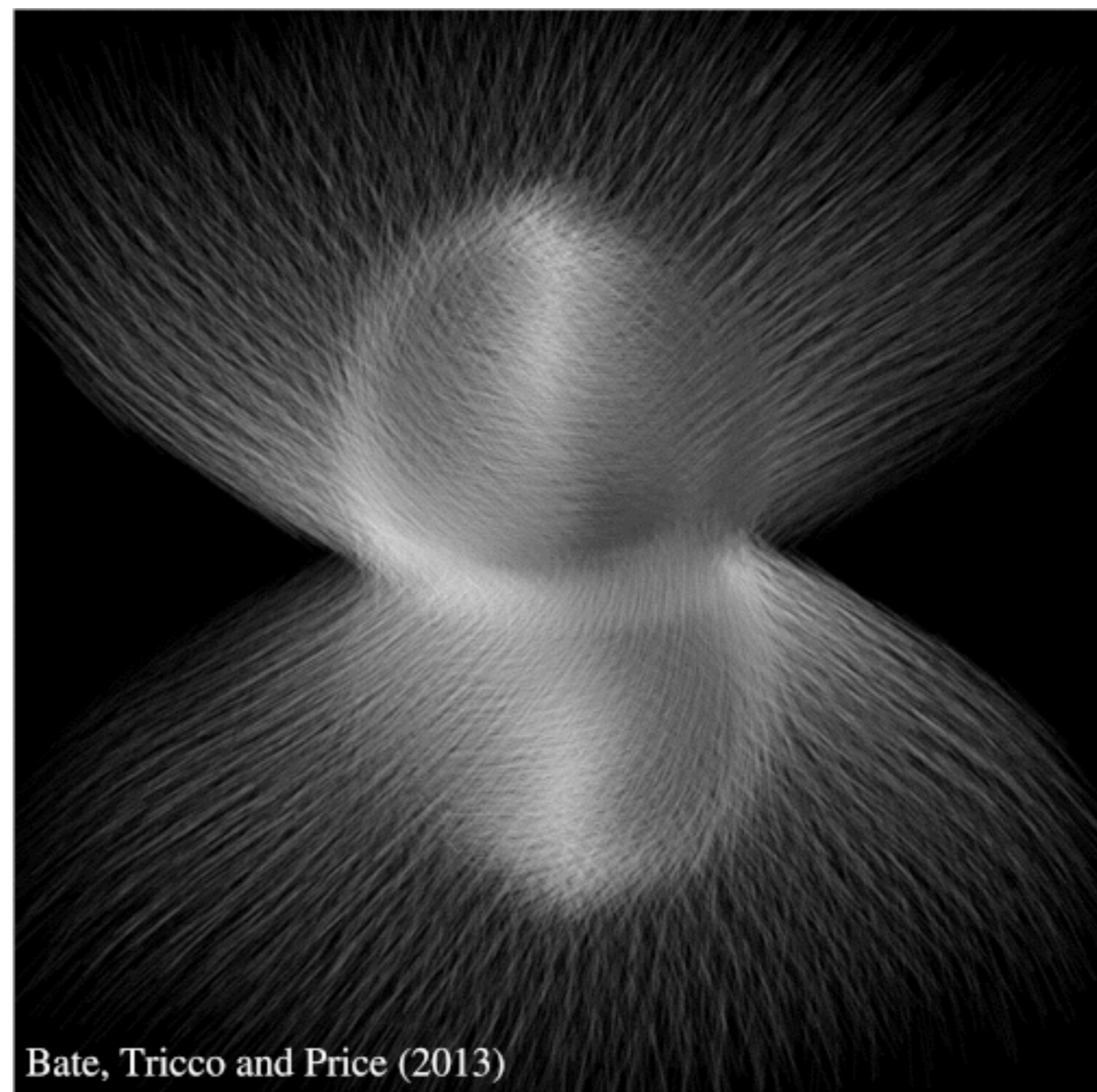
Performed with all dissipation, shock capturing and divergence cleaning turned on

MAGNETICALLY LAUNCHED OUTFLOWS

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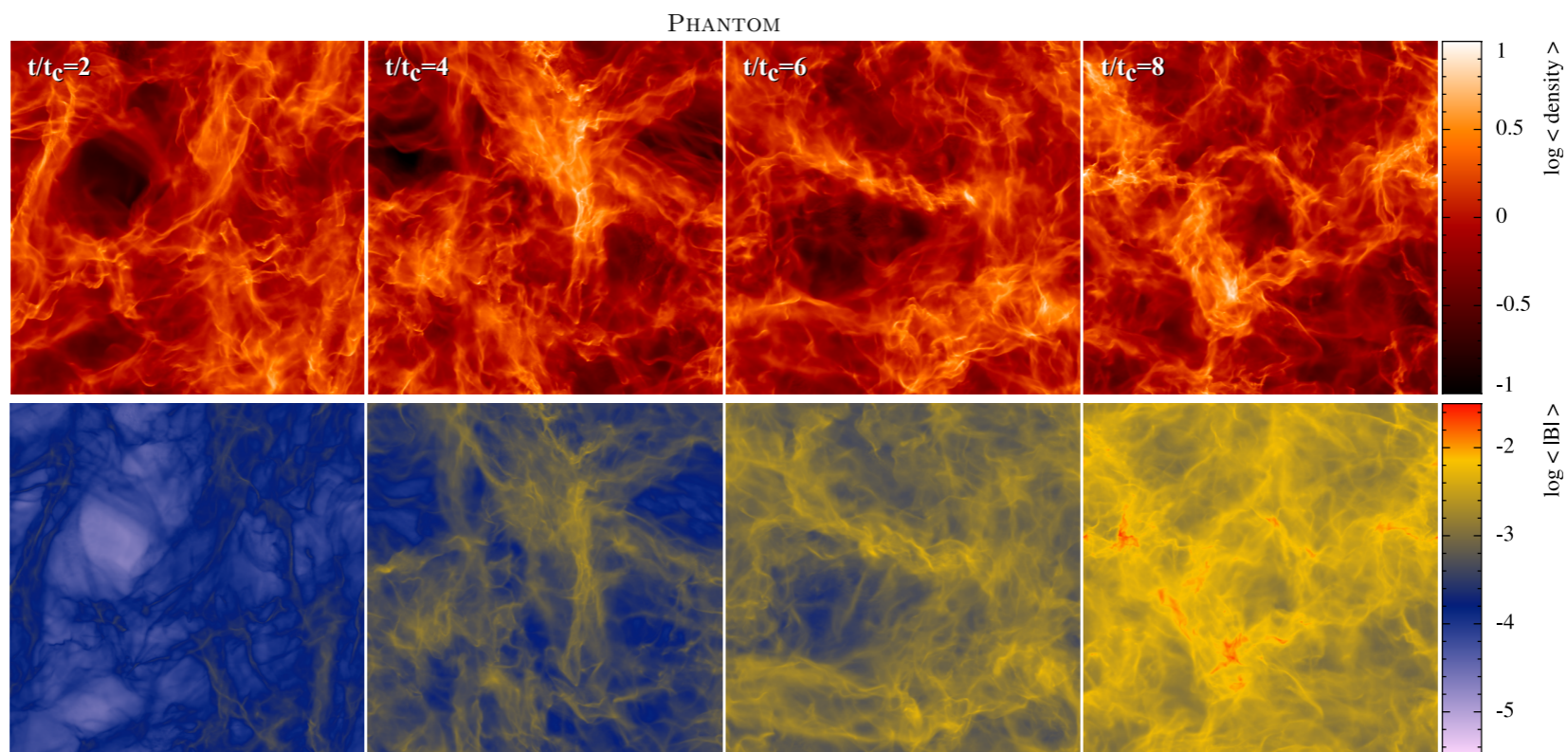
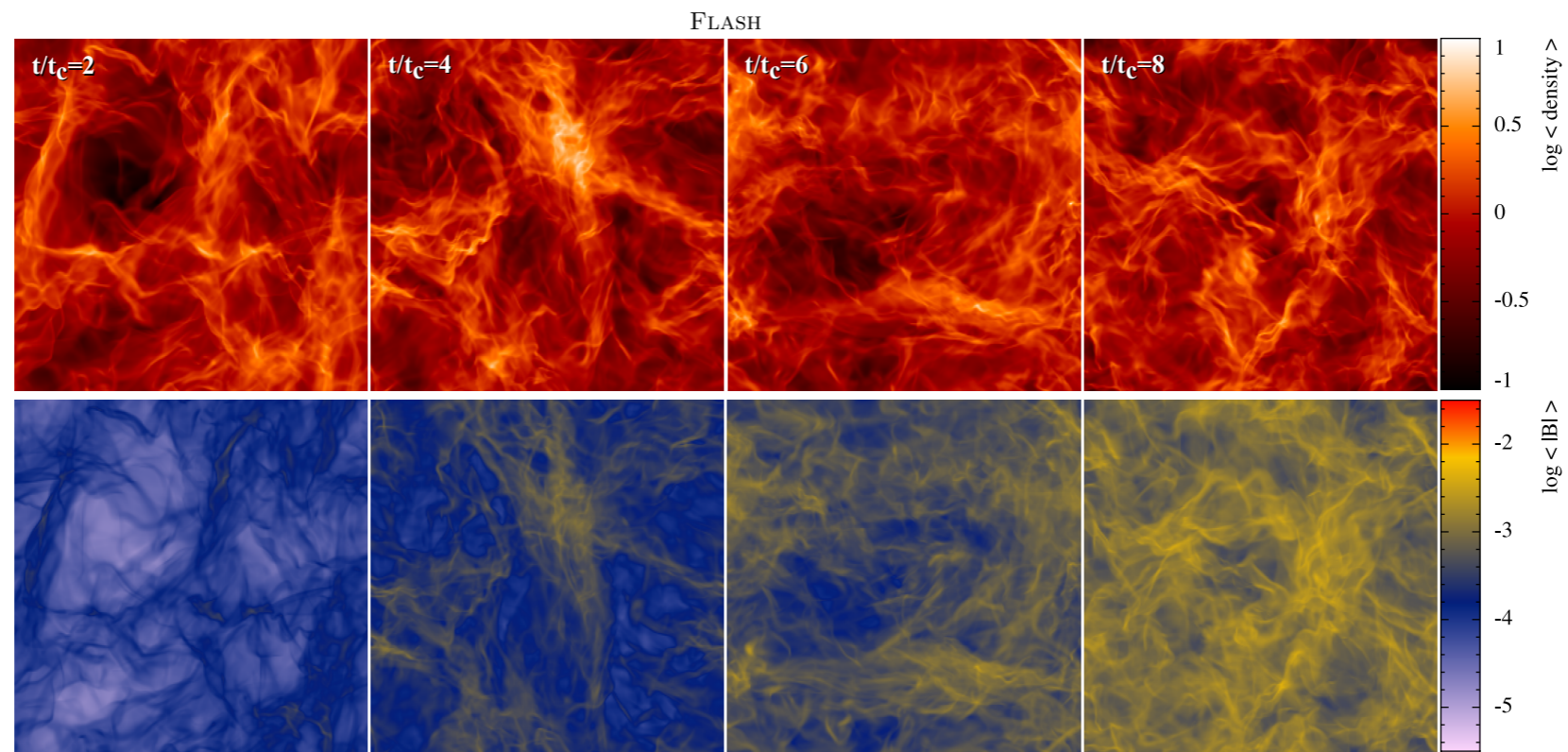
First core (100 x 100 au)



Second (protostellar) core (10 x 10 au)

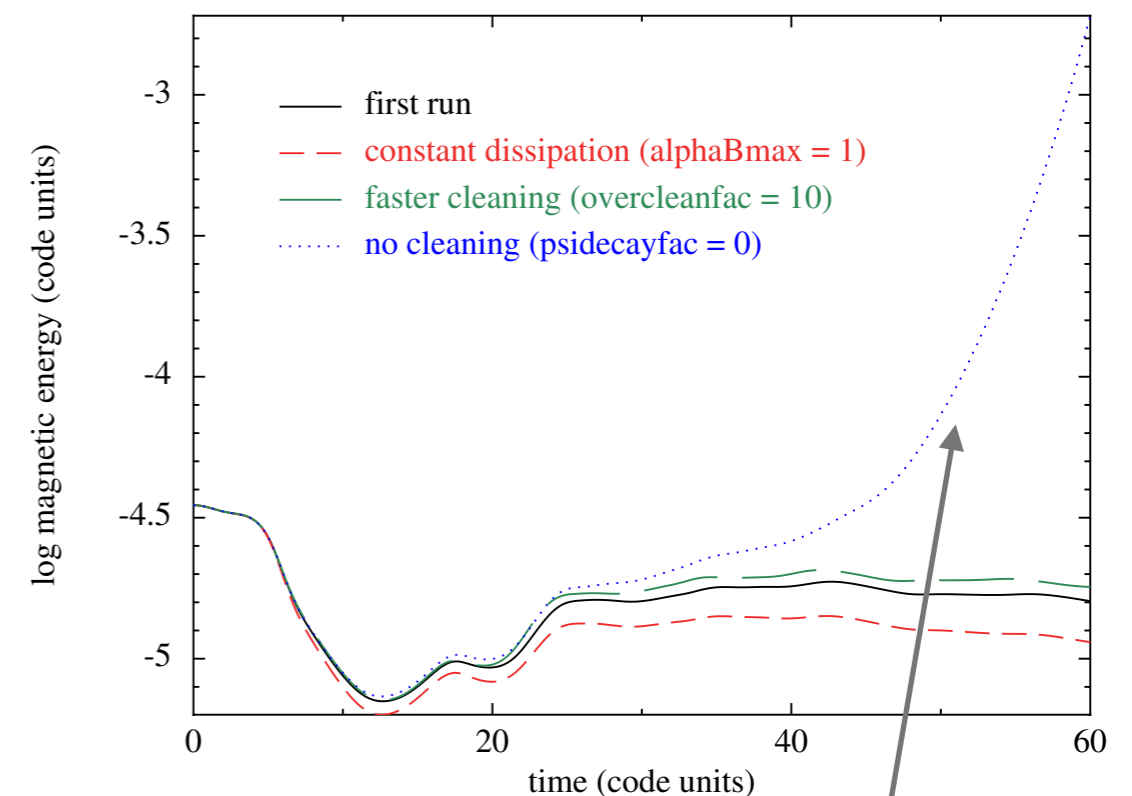
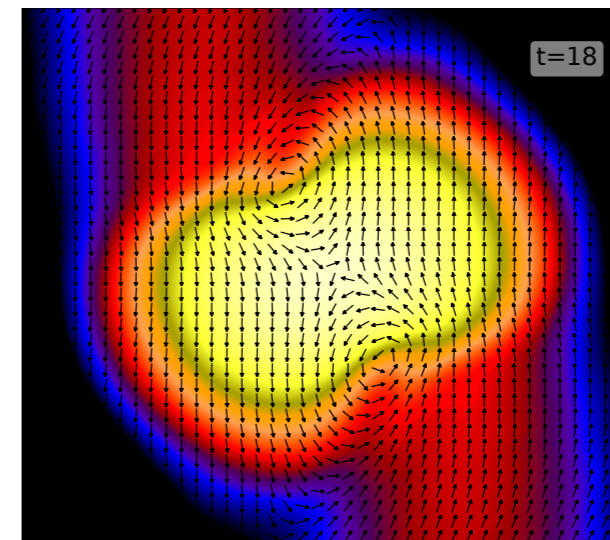
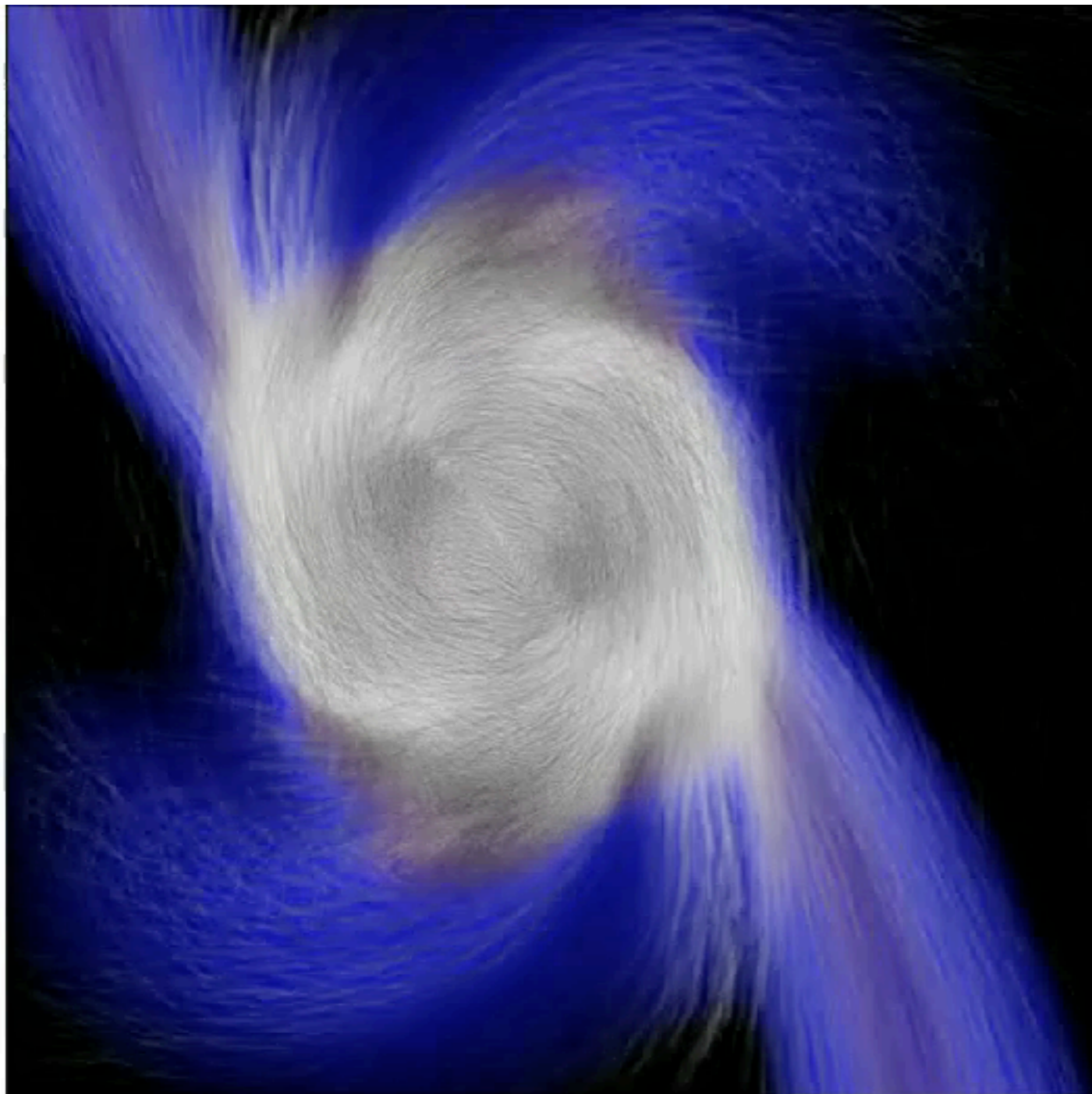
SMALL SCALE DYNAMO: FLASH VS PHANTOM

Tricco, Price & Federrath (2016)



MAGNETIC FIELDS IN TIDAL DISRUPTION EVENTS

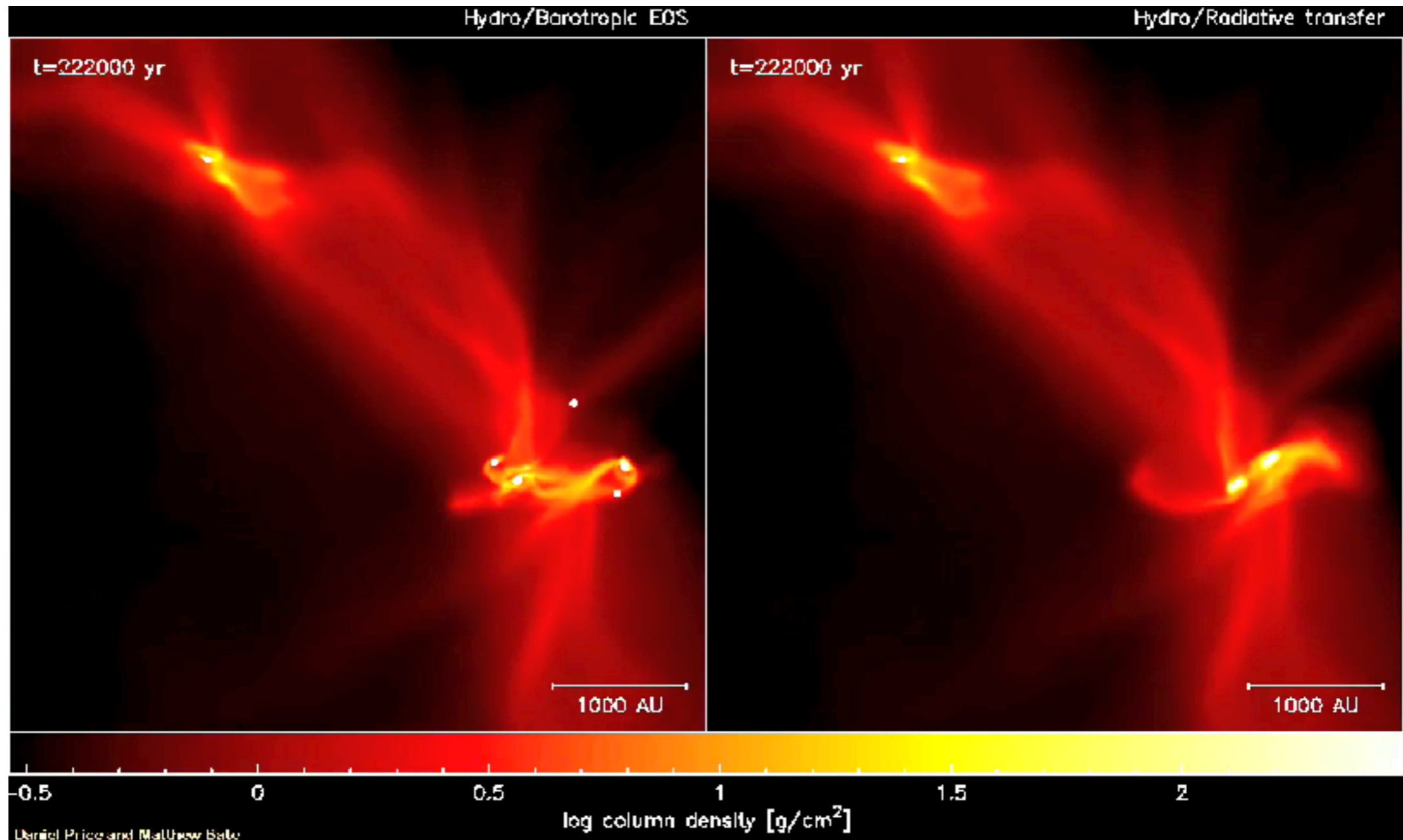
Bonnerot, Price, Rossi, Lodato
(2017), submitted to MNRAS



Danger! Can get artificial dynamo using “8 wave scheme” w/out div B cleaning

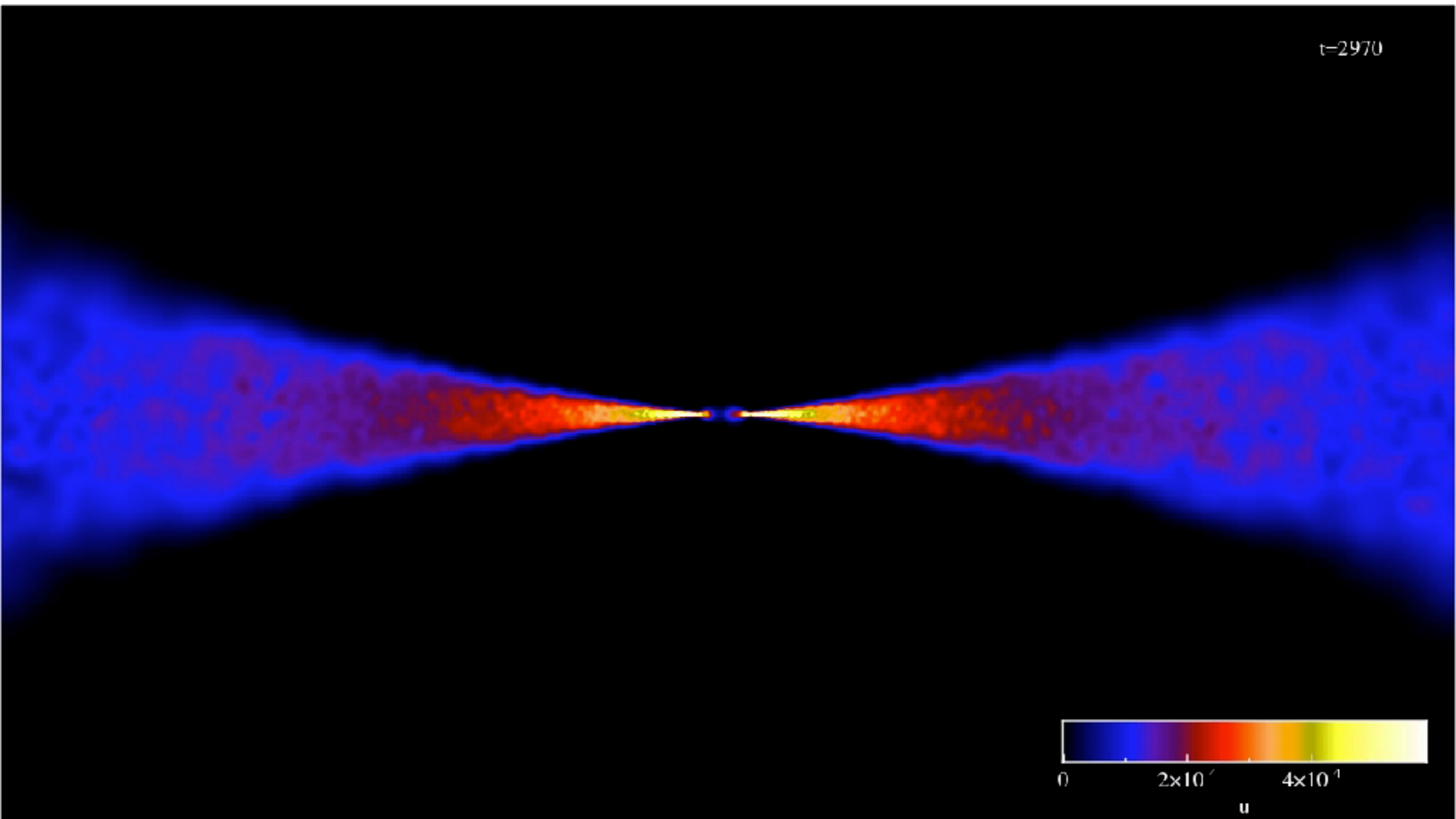
RADIATION

WHY RADIATION?



But Flux Limited Diffusion is both slow and wrong...

PHANTOM + MCFOST MONTE-CARLO RADIATION CODE



Mentiplay, Pinte & Price (2017), in prep.

SUMMARY

1. MHD in SPH is now fairly mature, useable out of the box for many practical applications
2. New one fluid dust method great for handling small grains / short stopping times
3. Direct coupling with Monte-Carlo radiation codes seems feasible, at least for disc studies

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Price et al. (2017), submitted to PASA
arXiv:1702.xxxx
<https://phantomsp.h.bitbucket.io>

PHANTOM: A smoothed particle hydrodynamics and

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**PHANTOM: A smoothed particle hydrodynamics and
magnetohydrodynamics code for astrophysics**

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