GRAND CHALLENGES IN COMPUTATIONAL ASTROPHYSICS: DUST, MHD AND RADIATION

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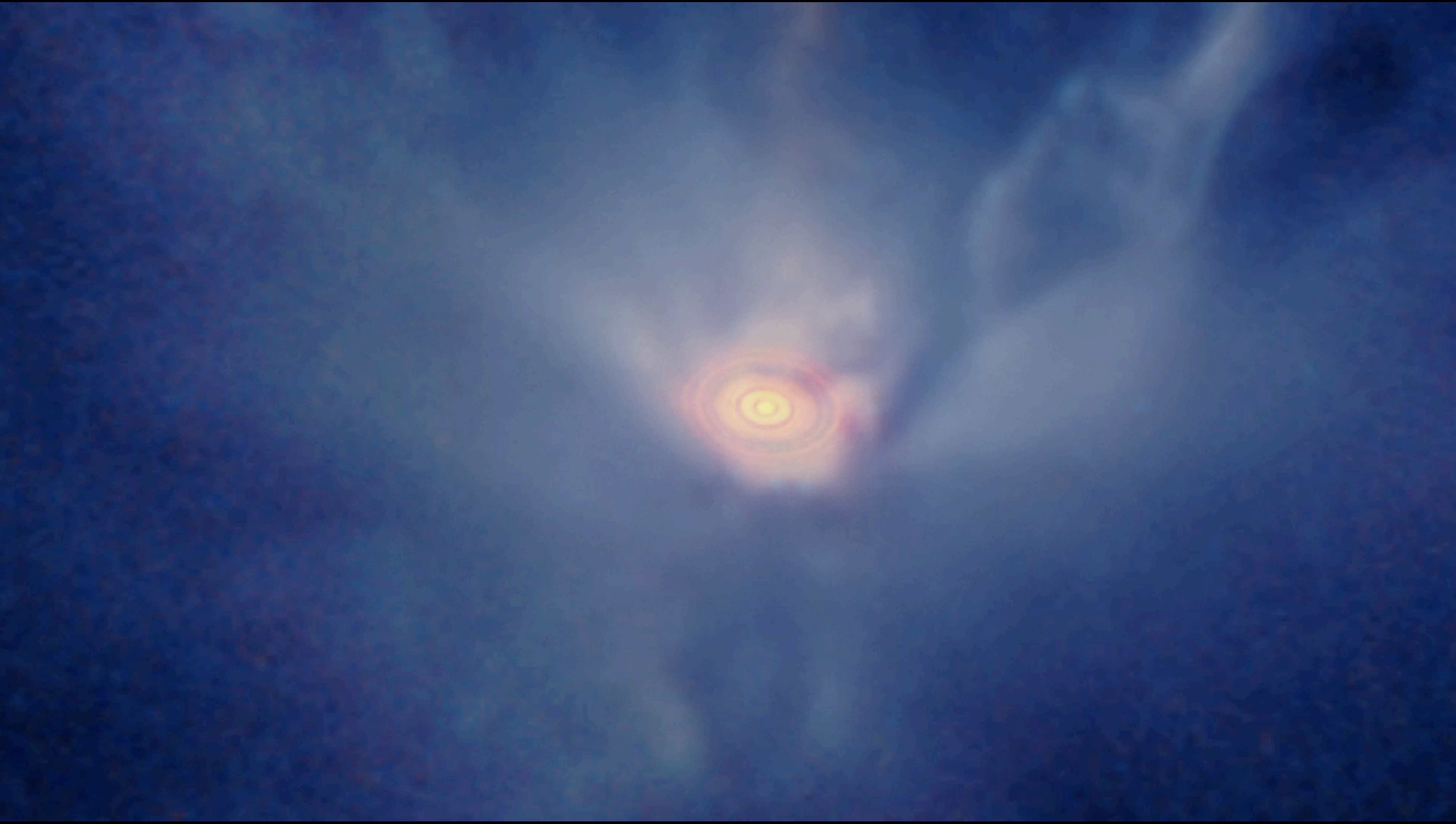
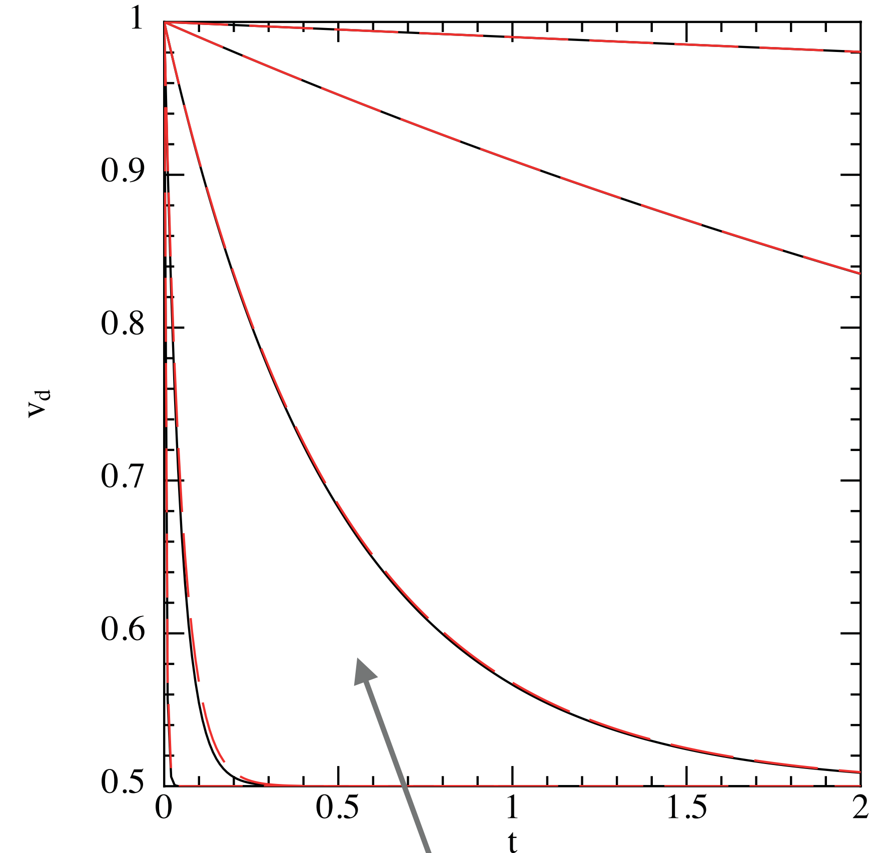
Carving through the codes: Challenges in Computational Astrophysics
12-17th February 2017, Davos, Switzerland



DUST

TWO FLUID



Characteristic timescale t_s

$$\frac{\partial \rho_\mathrm{g}}{\partial t} + \nabla.\left(\rho_\mathrm{g}\mathbf{v}_\mathrm{g}\right) = 0,$$

$$\frac{\partial \rho_\mathrm{d}}{\partial t} + \nabla.\left(\rho_\mathrm{d}\mathbf{v}_\mathrm{d}\right) = 0,$$

$$\frac{\partial \mathbf{v}_\mathrm{g}}{\partial t} + \left(\mathbf{v}_\mathrm{g}.\nabla\right)\mathbf{v}_\mathrm{g} = -\frac{\nabla P_\mathrm{g}}{\rho_\mathrm{g}} + K(\mathbf{v}_\mathrm{d} - \mathbf{v}_\mathrm{g}) + \mathbf{f},$$

$$\frac{\partial \mathbf{v}_\mathrm{d}}{\partial t} + \left(\mathbf{v}_\mathrm{d}.\nabla\right)\mathbf{v}_\mathrm{d} = -K(\mathbf{v}_\mathrm{d} - \mathbf{v}_\mathrm{g}) + \mathbf{f},$$

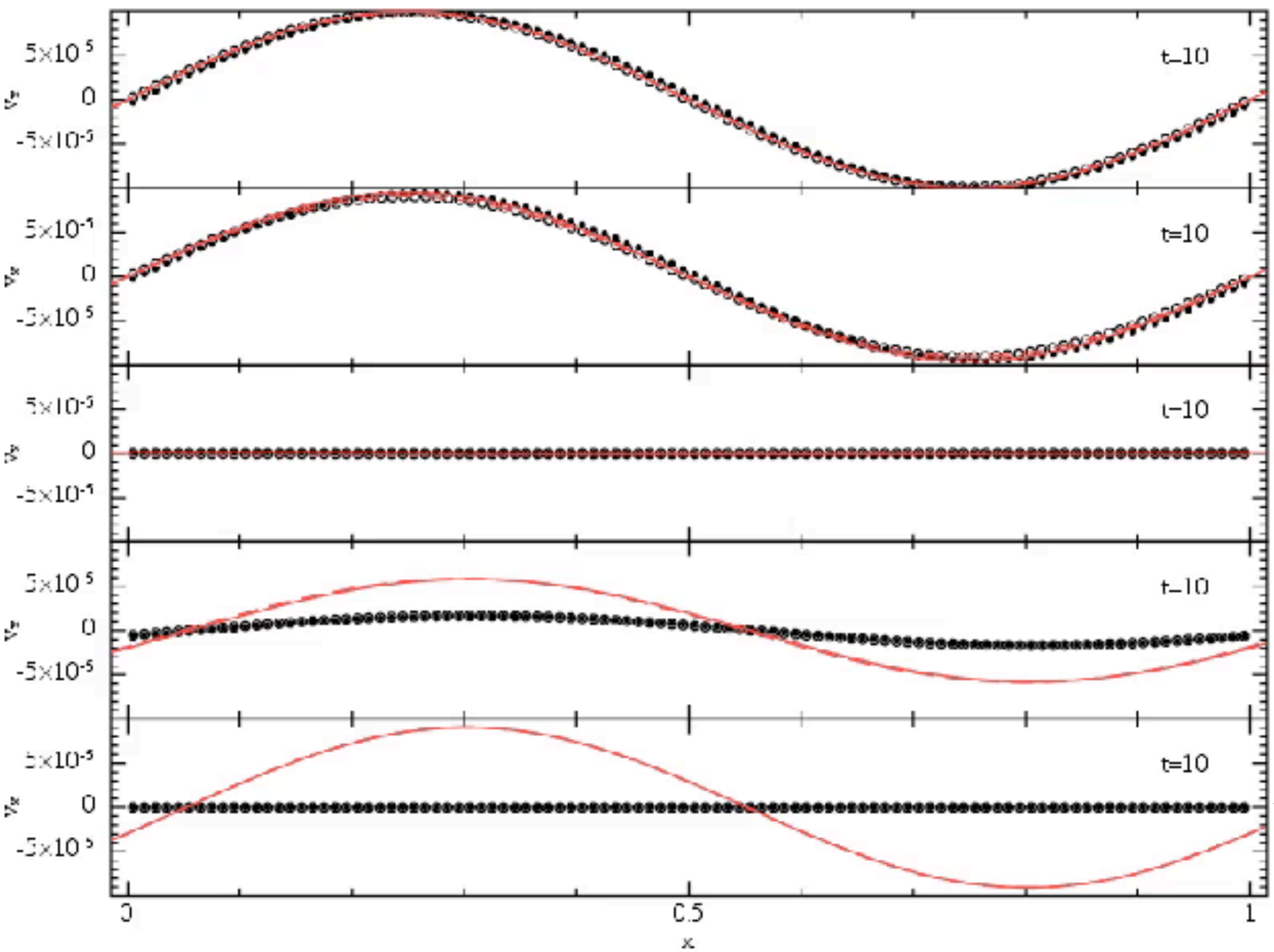
Timestep constraint: $\Delta t < t_\mathrm{s}$

DUSTYWAVE

Maddison (1993), Laibe & Price (2011)

$$(\omega^2 - c_{\rm s}^2 k^2) - \frac{i}{\omega t_{\rm s}}(\omega - \tilde{c}_{\rm s}^2 k^2) = 0$$

1) OVERDAMPING PROBLEM



No drag=no damping numerical=exact

Intermediate drag = strong damping in both numerical and exact

High drag =

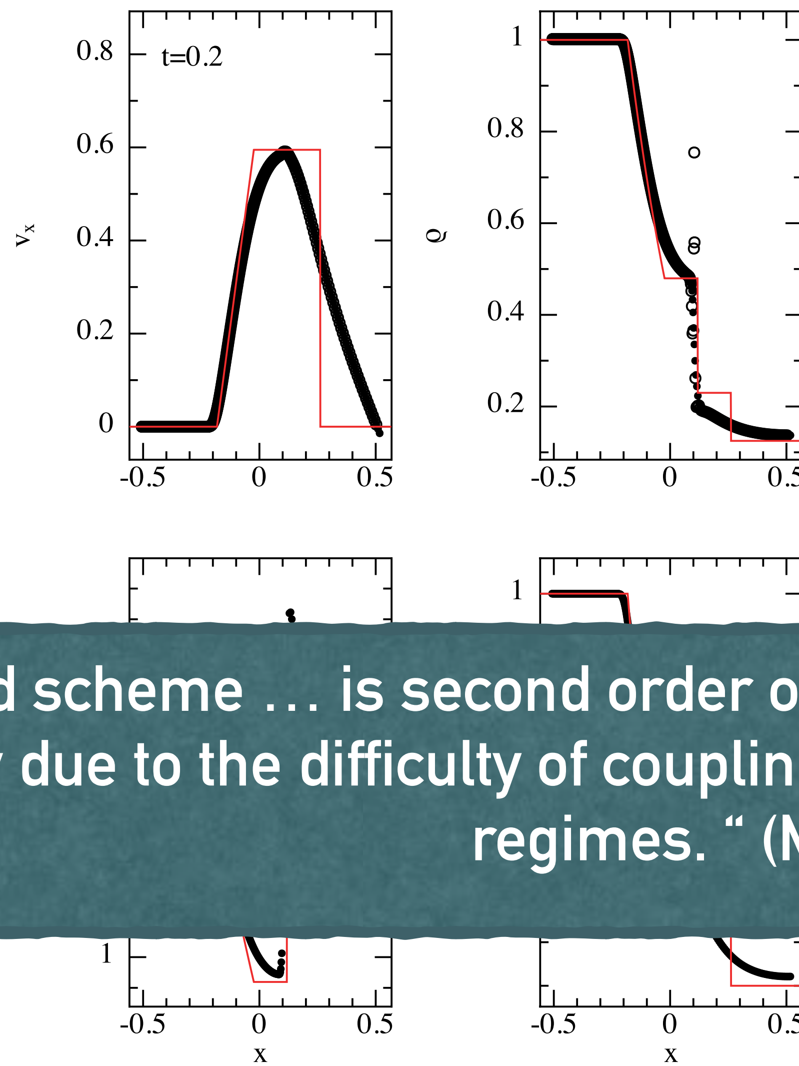
no damping

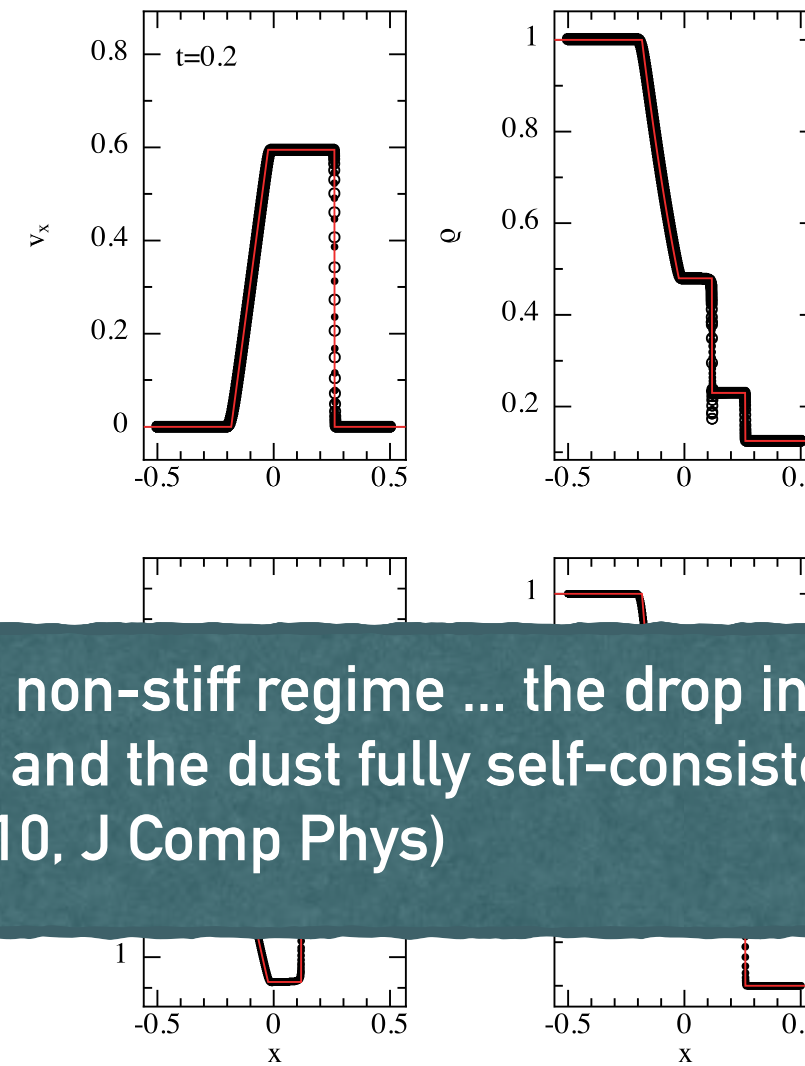
but numerical solution strongly damped

Red=analytic solution for dust/gas waves derived by Laibe & Price (2011) MNRAS 418, 1491

OVERDAMPING PROBLEM

Must resolve stopping length
L ~ cs ts





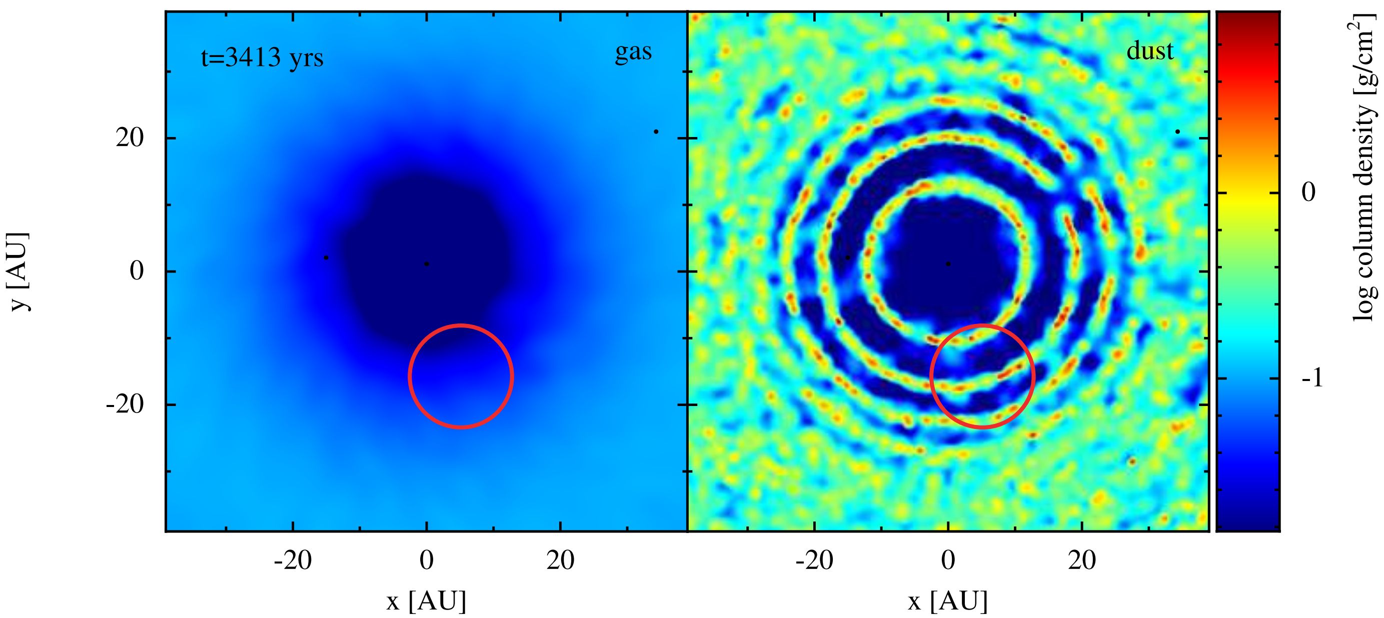
"The hybrid scheme … is second order only in the non-stiff regime ... the drop in accuracy … is most likely due to the difficulty of coupling the gas and the dust fully self-consistently in the stiff regimes. " (Miniati 2010, J Comp Phys)

sensible resolution

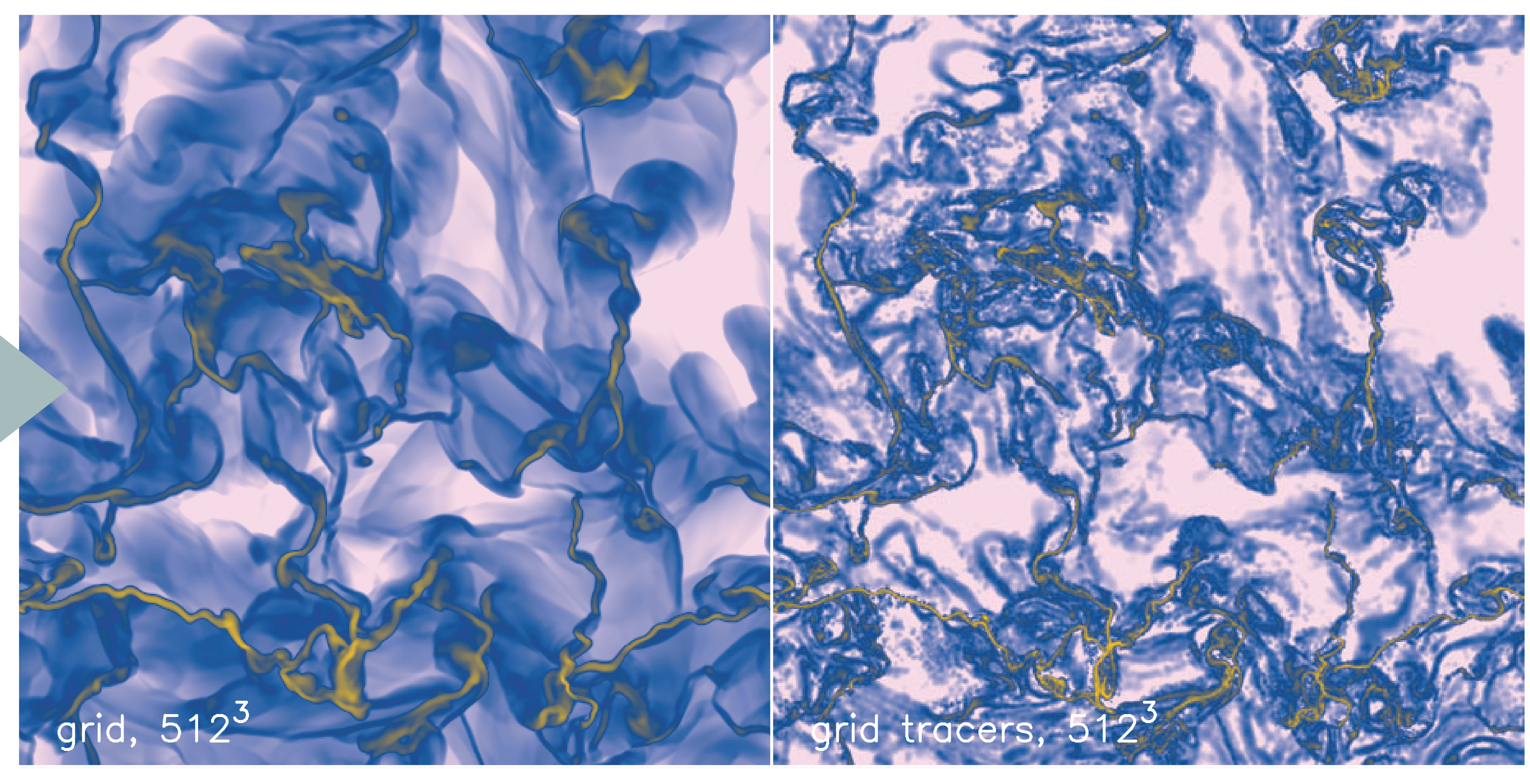
ludicrous resolution

II. DUST TRAPPING PROBLEM

- Dust particles get `stuck' below gas resolution length



Price & Federrath (2010): same issue for tracer/ dust particles on grids



DUST-GAS: ONE FLUID

One mixture with a differential velocity

$$\frac{\partial \rho_{\rm g}}{\partial t} + \nabla\cdot(\rho_{\rm g}\mathbf{v}_{\rm g}) = 0,$$

$$\frac{\partial \rho_{\rm d}}{\partial t} + \nabla\cdot(\rho_{\rm d}\mathbf{v}_{\rm d}) = 0,$$

$$\frac{\partial \mathbf{v}_{\rm g}}{\partial t} + (\mathbf{v}_{\rm g}\cdot\nabla)\mathbf{v}_{\rm g} = -\frac{\nabla P_{\rm g}}{\rho_{\rm g}} + \frac{K}{\rho_{\rm g}}(\mathbf{v}_{\rm d} - \mathbf{v}_{\rm g}) + \mathbf{f},$$

$$\frac{\partial \mathbf{v}_{\rm d}}{\partial t} + (\mathbf{v}_{\rm d}\cdot\nabla)\mathbf{v}_{\rm d} = -\frac{K}{\rho_{\rm d}}(\mathbf{v}_{\rm d} - \mathbf{v}_{\rm g}) + \mathbf{f},$$

$$\frac{{\rm d}\rho}{{\rm d}t} = -\rho(\nabla\cdot\mathbf{v}),$$

$$\frac{{\rm d}\epsilon}{{\rm d}t} = -\frac{1}{\rho}\nabla\cdot\left[\epsilon(1-\epsilon)\rho\Delta\mathbf{v}\right],$$

$$\frac{{\rm d}\mathbf{v}}{{\rm d}t} = -\frac{\nabla P}{\rho} - \frac{1}{\rho}\nabla\cdot\left[\epsilon(1-\epsilon)\rho\Delta\mathbf{v}\Delta\mathbf{v}\right] + \mathbf{f},$$

$$\frac{{\rm d}\Delta\mathbf{v}}{{\rm d}t} = -\frac{\Delta\mathbf{v}}{t_{\rm s}} + \frac{\nabla P}{\rho_{\rm g}} - (\Delta\mathbf{v}\cdot\nabla)\mathbf{v} + \frac{1}{2}\nabla\left[(2\epsilon - 1)\Delta\mathbf{v}^2\right].$$

$$\rho = \rho_{\rm d} + \rho_{\rm g}$$

$$\epsilon = \rho_{\rm d}/\rho$$

$$\mathbf{v} = \frac{\rho_{\rm d}\mathbf{v}_{\rm d} + \rho_{\rm g}\mathbf{v}_{\rm g}}{\rho}$$

$$\Delta\mathbf{v} = \mathbf{v}_{\rm d} - \mathbf{v}_{\rm g}$$

Laibe & Price (2014) MNRAS 440, 2136

ONE FLUID FOR SMALL GRAINS

Laibe & Price (2014), Price & Laibe (2015)

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho(\nabla \cdot \boldsymbol{v})$$

$$\frac{\mathrm{d}\epsilon}{\mathrm{d}t} = -\frac{1}{\rho}\nabla \cdot \left[\epsilon(1-\epsilon)\rho\Delta\boldsymbol{v}\right]$$

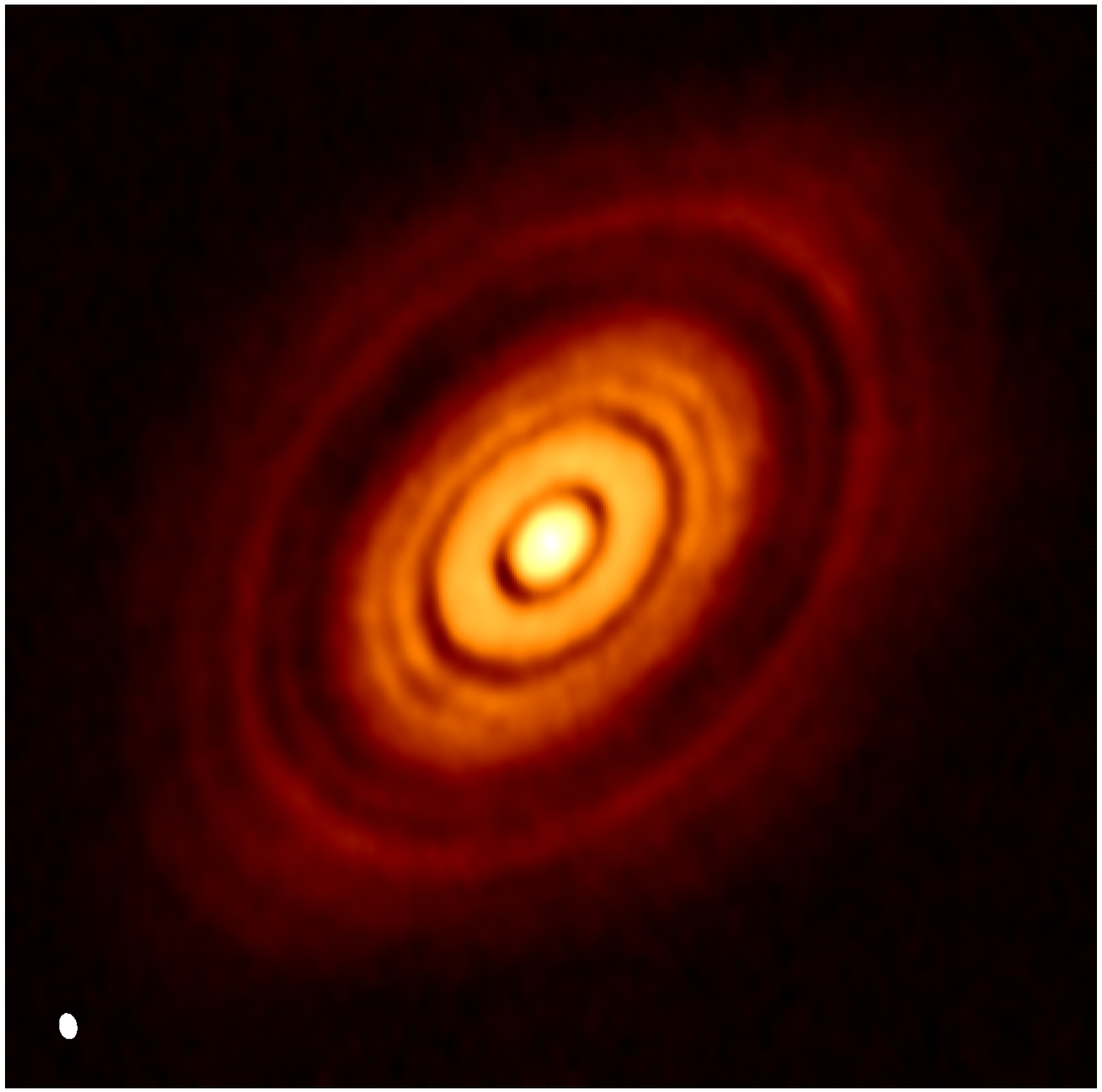
$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = -\frac{\nabla P}{\rho} - \frac{1}{\rho}\nabla \cdot \left[\epsilon(1-\epsilon)\Delta\boldsymbol{v}\Delta\boldsymbol{v}\right]$$

$$\frac{\mathrm{d}\Delta\boldsymbol{v}}{\mathrm{d}t} = -\frac{\Delta\boldsymbol{v}}{t_{\mathrm{s}}} + \frac{\nabla P}{\rho_{\mathrm{g}}} - (\Delta\boldsymbol{v}\cdot\nabla)\boldsymbol{v} + \frac{1}{2}\nabla\left[(2\epsilon-1)\Delta\boldsymbol{v}^2\right]$$

EXPLICIT when stopping time is short

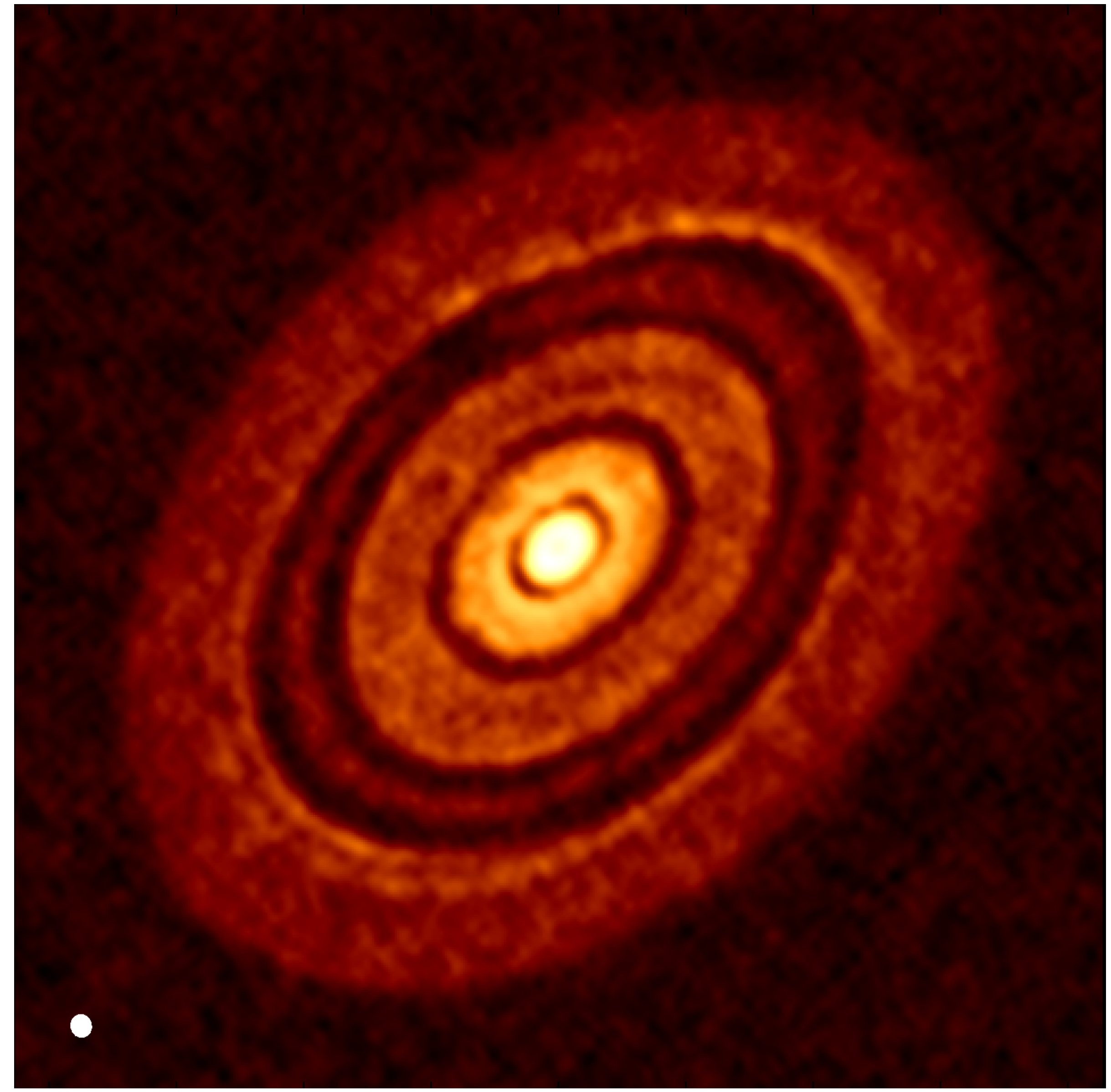
Terminal velocity approximation: See also Youdin & Goodman (2005); Chiang (2008); Barranco 2009, Jacquet et al. 2011

APPLICATION TO HL TAU



Observed image

ALMA partnership et al. (2015)



Our simulation

Dipierro, Price, Laibe, Hirsh, Cerioli & Lodato (2015)

EXAMPLE II: "HORSESHOES" IN TRANSITION DISCS *Ragusa+2017*

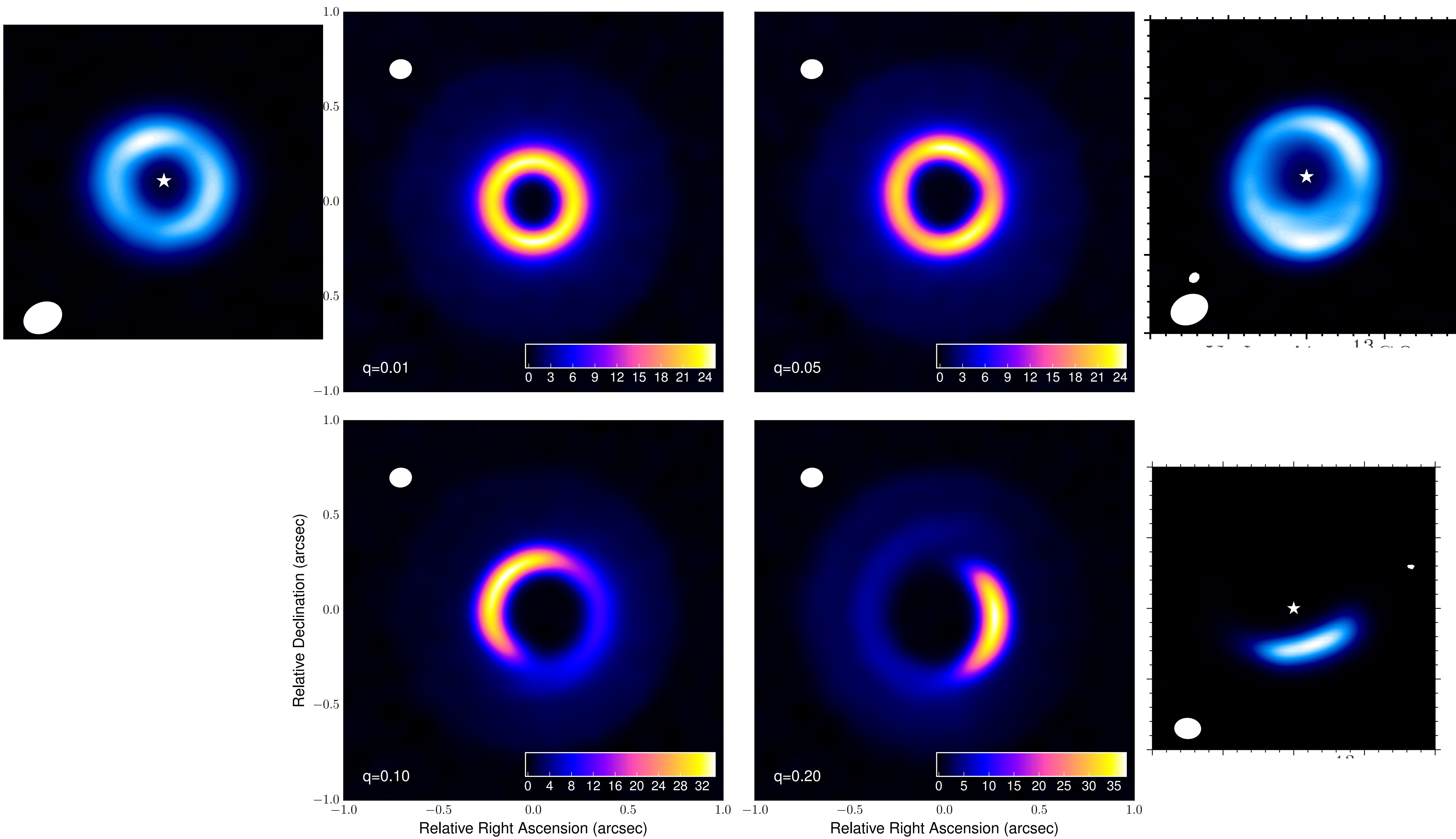


Figure 2. Comparison of ALMA simulated observations at 345 GHz of disc models with a mass ratio $q = 0.01$ (upper left), $q = 0.05$ (upper right), $q = 0.1$ (bottom left) and $q = 0.2$ (bottom right). Intensities are in mJy beam^{-1}. The white colour in the filled ellipse in the upper left corner indicates the size of the half-power contour of the synthesized beam: 0.12×0.1 arcsec ($\sim 16 \times 13$ au at 130 pc.).

MHD

SMOOTHED PARTICLE MAGNETOHYDRODYNAMICS

see review by Price (2012) J. Comp. Phys. 231, 759

$$L = \int \left(\frac{1}{2}\rho v^2 - \rho u - \frac{1}{2\mu_0} B^2 \right) \mathrm{d}V$$

$$L = \sum_a m_a \left(\frac{1}{2} v_a^2 - u_a - \frac{B_a^2}{2\mu_0 \rho_a} \right)$$

Price & Monaghan (2004a,b, 2005)

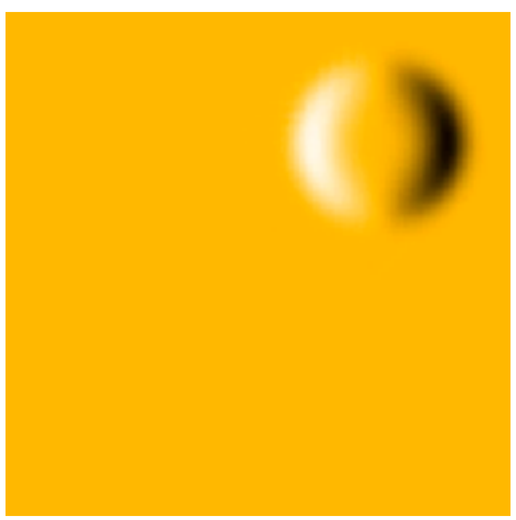
Euler-Lagrange equations give discrete form of:

$$\frac{\mathrm{d}\boldsymbol{\rho}}{\mathrm{d}t} = -\boldsymbol{\rho}(\nabla \cdot \mathbf{v})$$

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\frac{1}{\rho}\nabla \cdot \left[\left(P + \frac{1}{2}\frac{B^2}{\mu_0} \right) \mathbf{I} - \frac{\mathbf{BB}}{\mu_0} \right] - \frac{\mathbf{B}(\nabla \cdot \mathbf{B})}{\mu_0 \rho}$$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{P}{\rho}(\nabla \cdot \mathbf{v})$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathbf{B}}{\rho} \right) = \left(\frac{\mathbf{B}}{\rho} \cdot \nabla \right) \mathbf{v}$$



Divergence advection test from Dedner et al. (2002)

Subtract div B source term for stability

Dissipationless: Must add dissipation terms to handle shocks and discontinuities

Need to separately handle div B = 0

These equations are equivalent to the "8-wave formulation" of Powell et al. 1994

HYPERBOLIC/PARABOLIC DIVERGENCE CLEANING

Dedner et al. (2002)
Price & Monaghan (2005)
Mignone & Tzeferacos (2010)

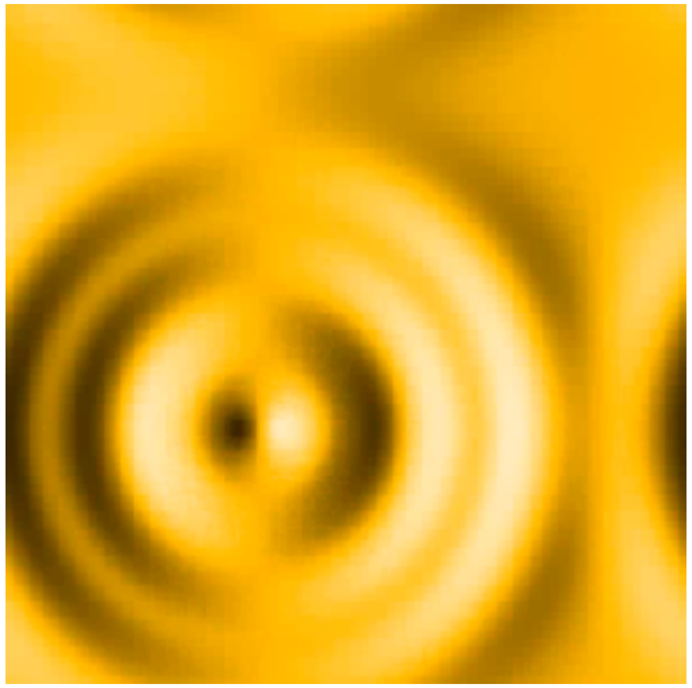
$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} = -\nabla\psi$$

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = -c_h^2(\nabla\cdot\mathbf{B}) - \frac{\sigma c_h}{h}\psi$$

Hyperbolic

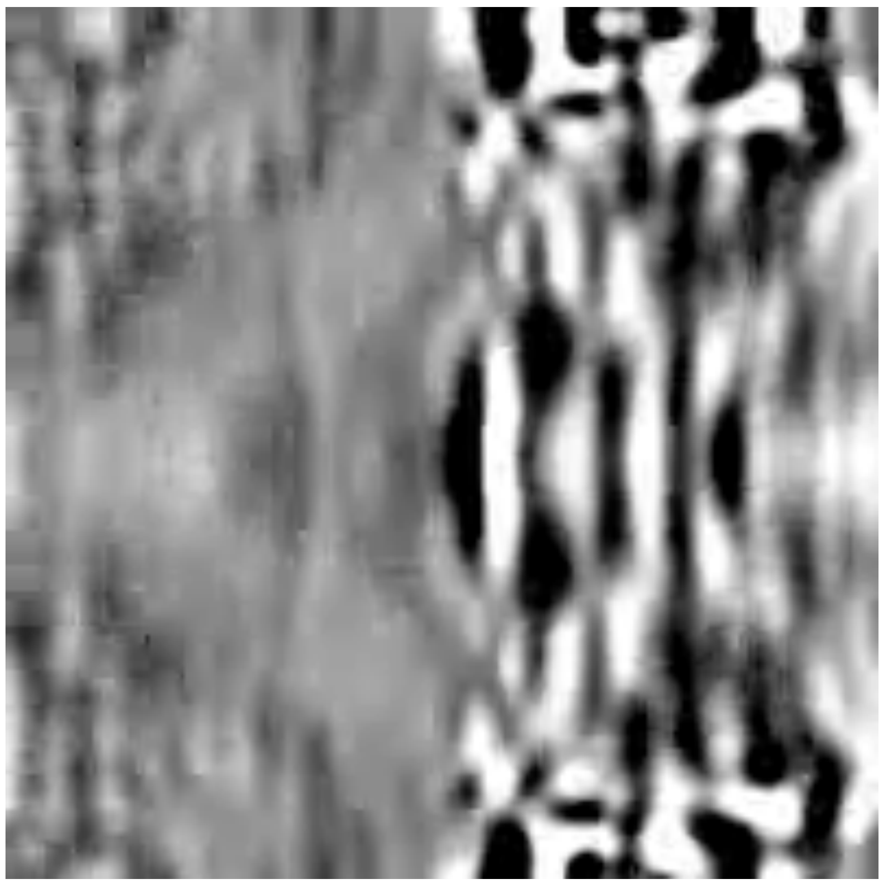
Parabolic

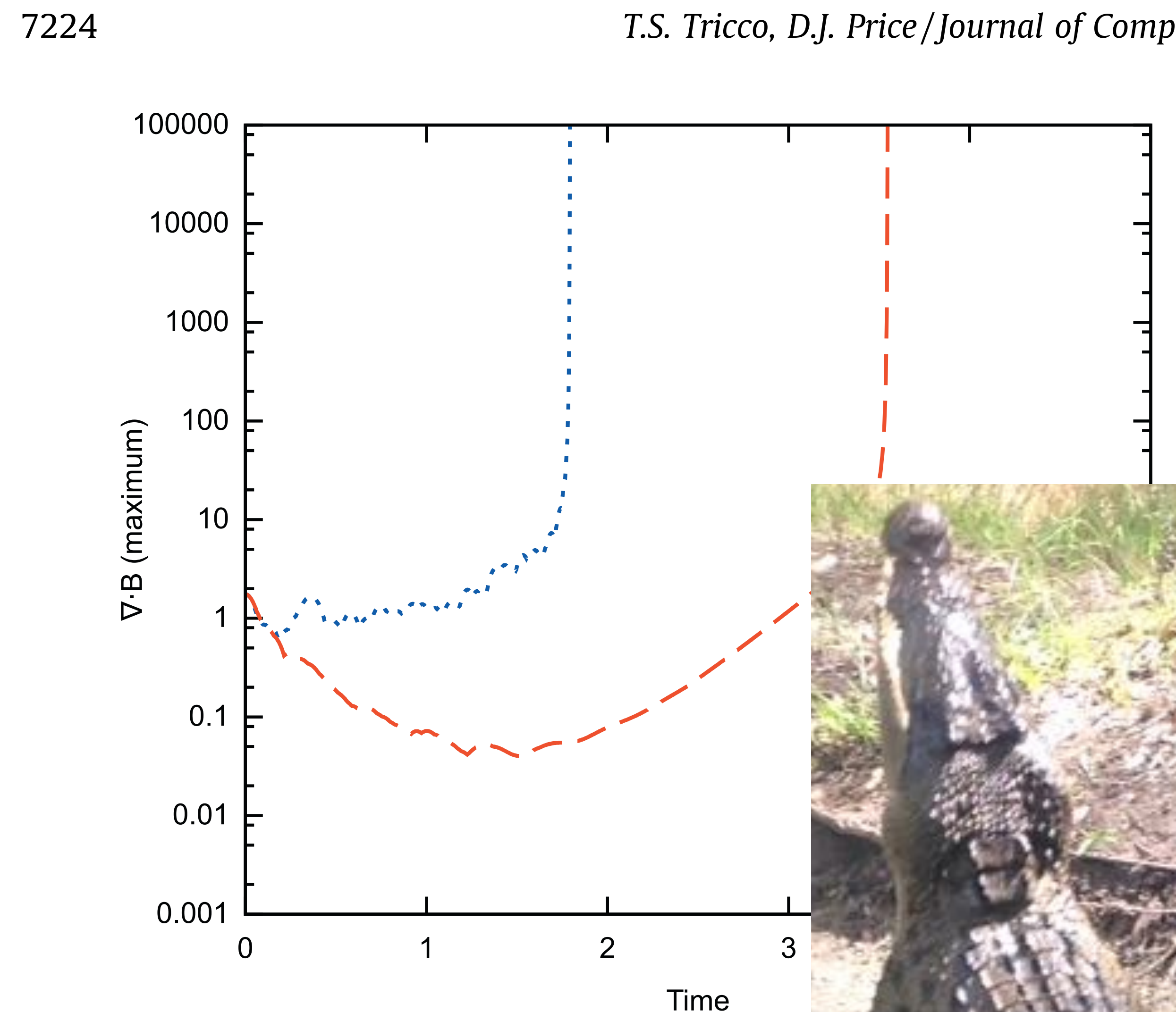
$$\frac{1}{c_h^2}\frac{\partial^2(\nabla\cdot\mathbf{B})}{\partial t^2} + \nabla^2(\nabla\cdot\mathbf{B}) + \frac{1}{\lambda c_h}\frac{\partial(\nabla\cdot\mathbf{B})}{\partial t} = 0$$



Hyperbolic term only

WHEN CLEANING ATTACKS





Divergence advection test (Dedner et al. 2002) with 10:1 jump in density

"CONSTRAINED" HYPERBOLIC/PARABOLIC DIVERGENCE CLEANING

Tricco & Price (2012); Tricco, Price & Bate (2016)

- Define energy associated with cleaning field

$$E = \int \left[\frac{1}{2}\frac{B^2}{\mu_0} + \frac{1}{2}\frac{\psi^2}{\mu_0 c_h^2} \right] \mathrm{dV}$$

- Enforce energy conservation in hyperbolic terms

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \int \left[\frac{\mathbf{B}}{\mu_0} \cdot \left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} \right)_{\psi} + \frac{\psi}{\mu_0 c_{\mathrm{h}}^2}\frac{\mathrm{d}\psi}{\mathrm{d}t} - \frac{\psi^2}{2\mu_0 \rho c_{\mathrm{h}}^2}\frac{\mathrm{d}\rho}{\mathrm{d}t} - \frac{\psi^2}{\mu_0 c_{\mathrm{h}}^3}\frac{\mathrm{d}c_{\mathrm{h}}}{\mathrm{d}t} \right] \mathrm{d}V = 0$$

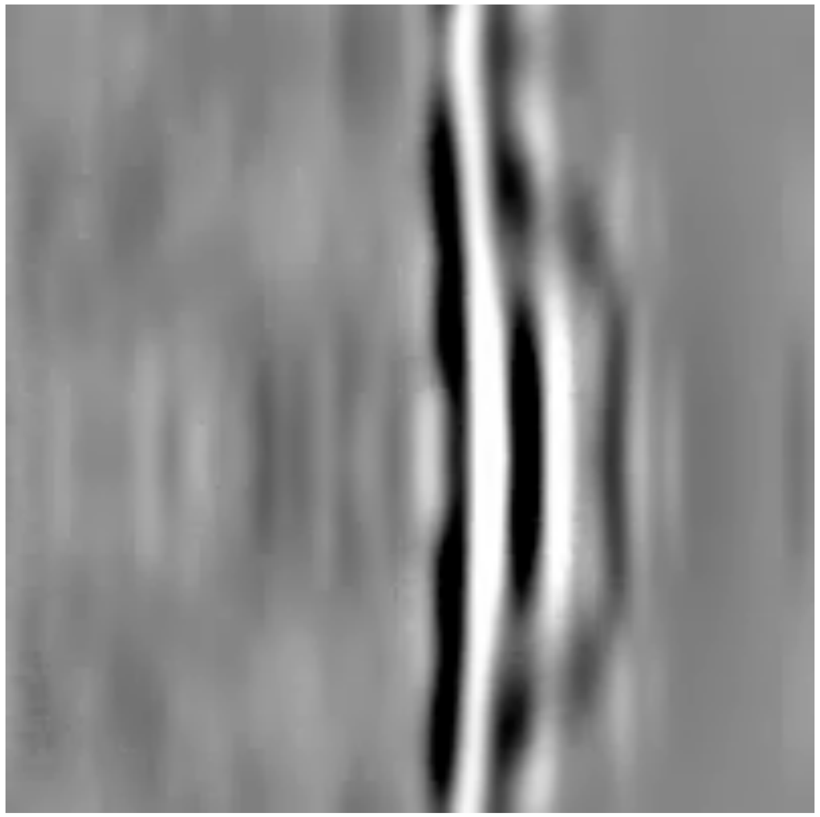
$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} = -\nabla\psi$$

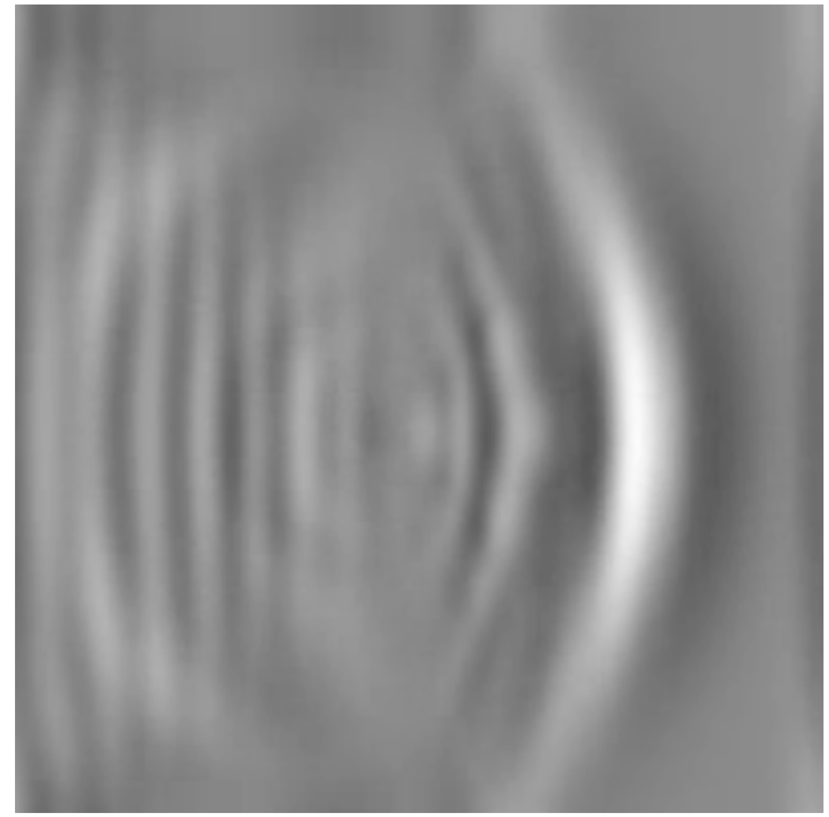
$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = -c_h^2(\nabla \cdot \mathbf{B}) - \frac{\sigma c_h}{h}\psi - \frac{1}{2}\psi(\nabla \cdot \mathbf{v})$$

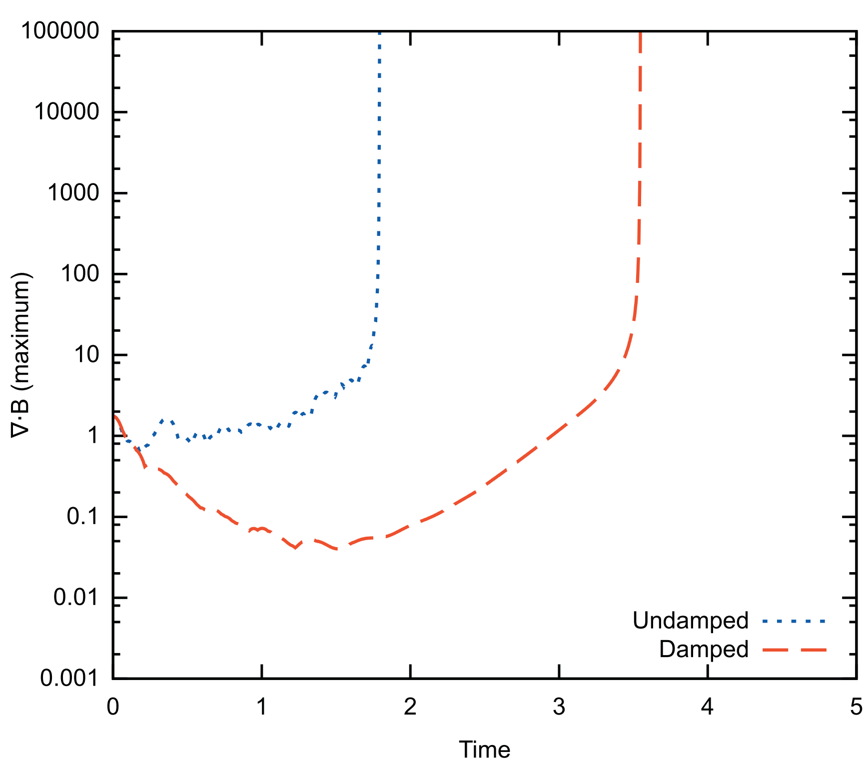
Requires particular choice of operators here

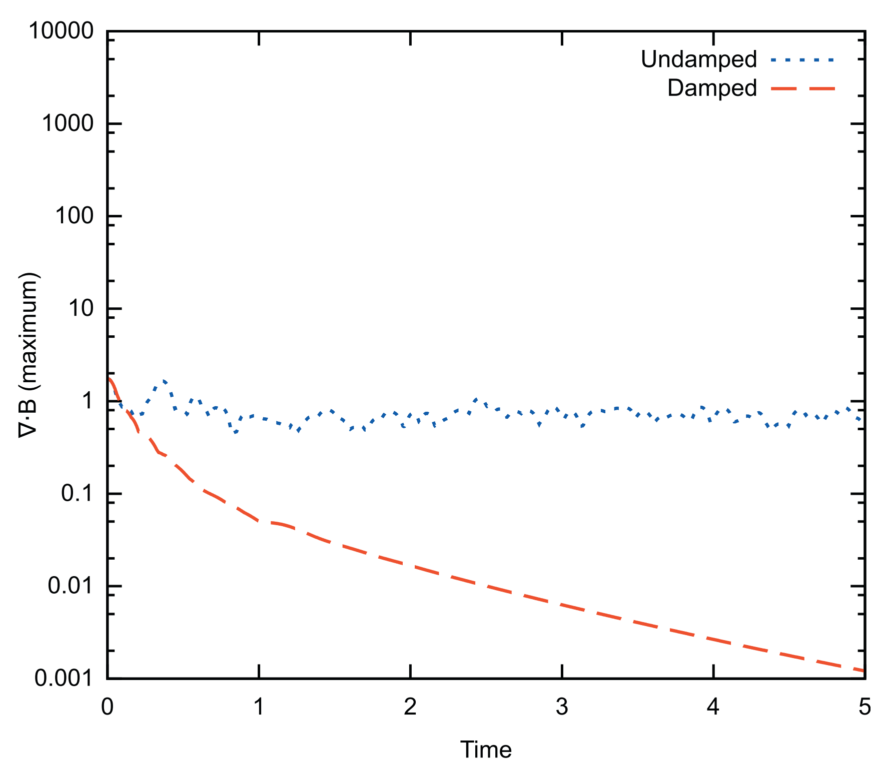
- Can enforce exact energy conservation in SPH discretisation

CONSTRAINED HYPERBOLIC/PARABOLIC CLEANING





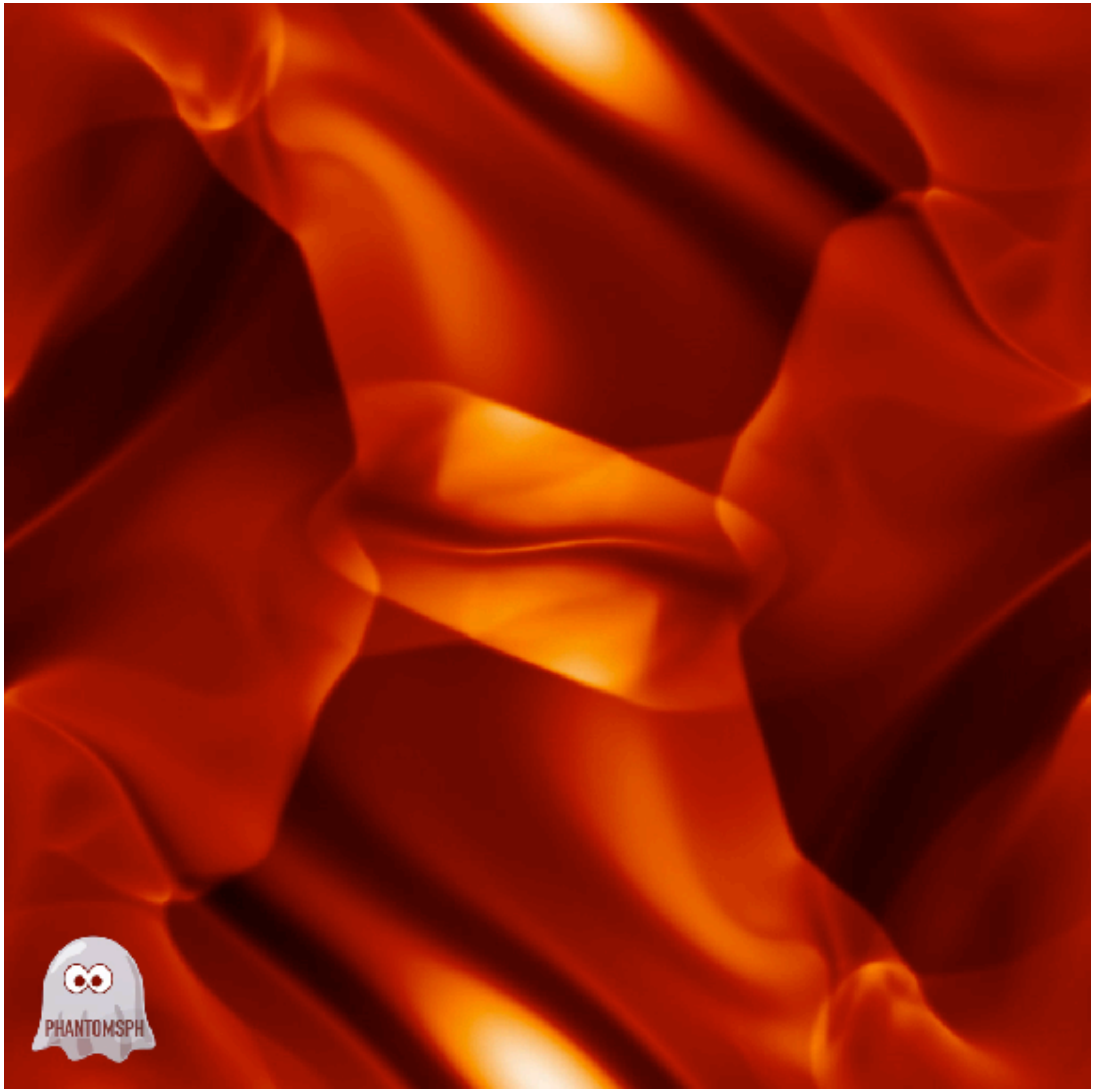


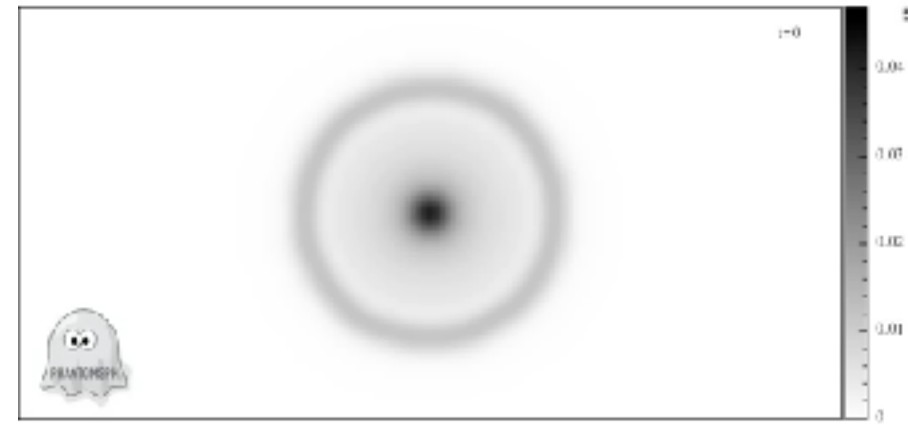


Parabolic term is negative definite!

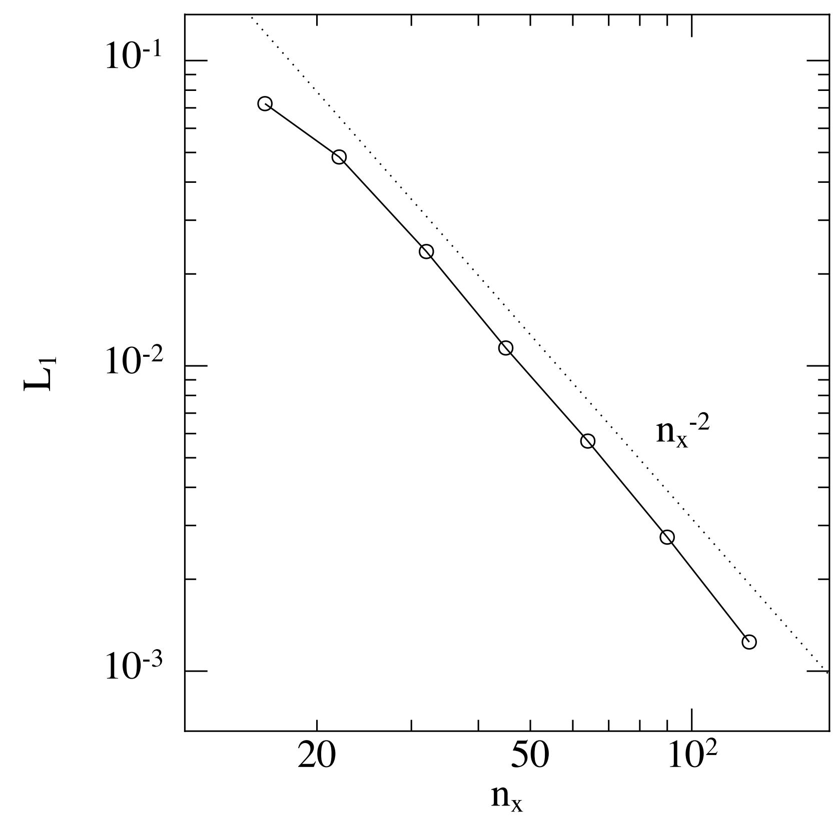
PHANTOM SPMHD CODE

Price et al. (2017)





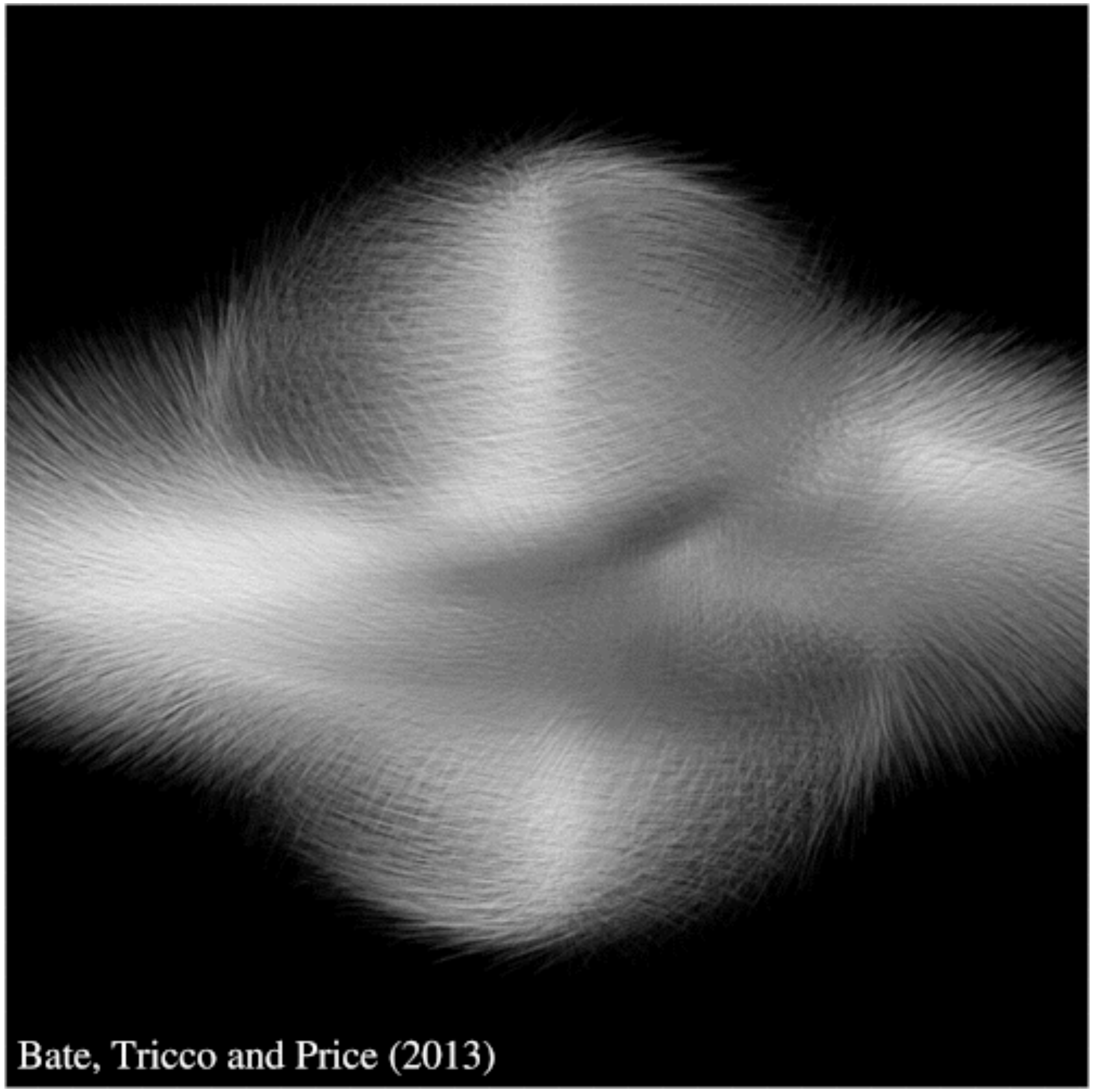
Advection of current loop (Gardiner & Stone 2005, 2008)



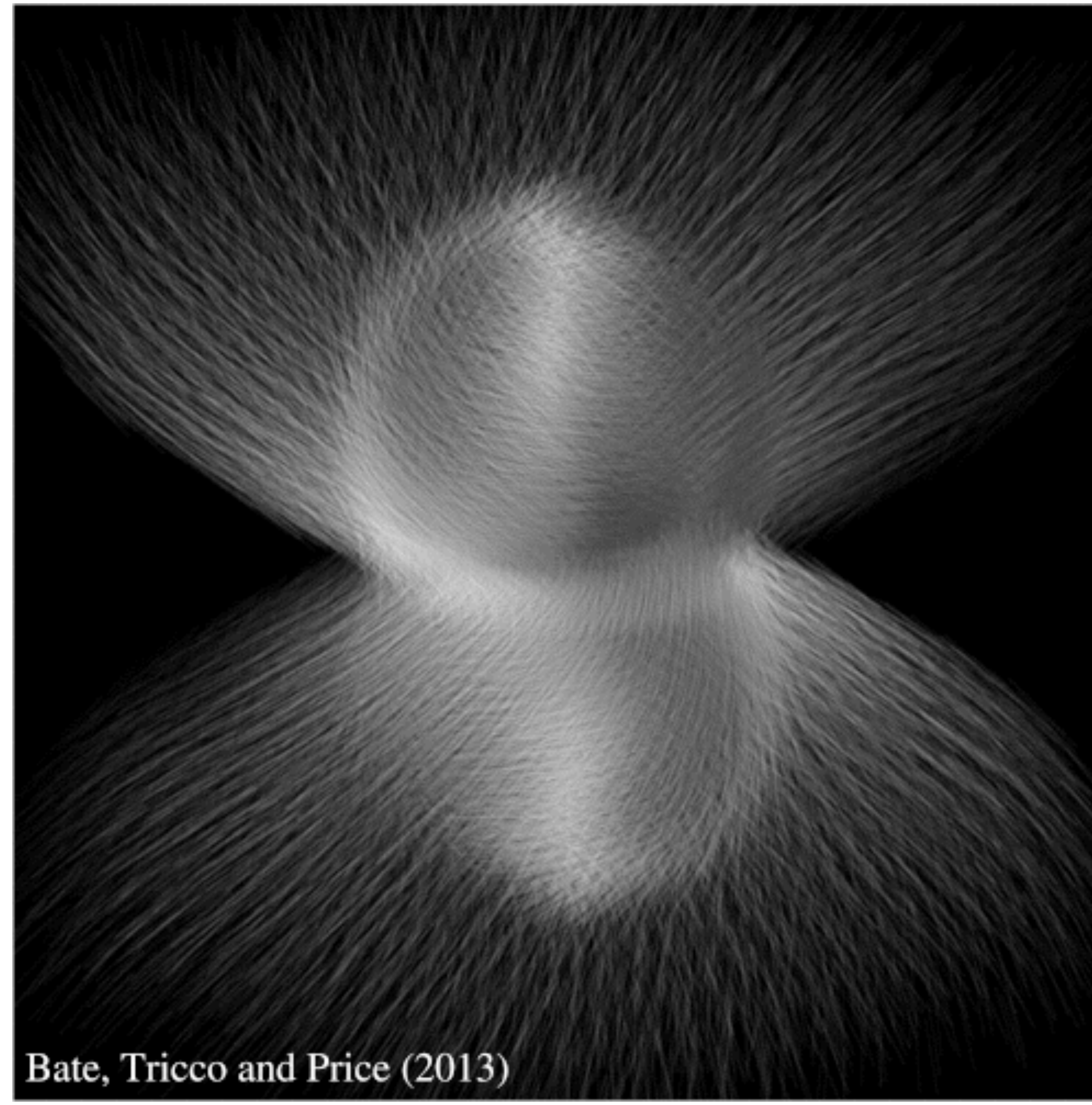
Convergence on circularly polarised Alfvén wave with ALL dissipation turned on

Performed with all dissipation, shock capturing and divergence cleaning turned on

MAGNETICALLY LAUNCHED OUTFLOWS



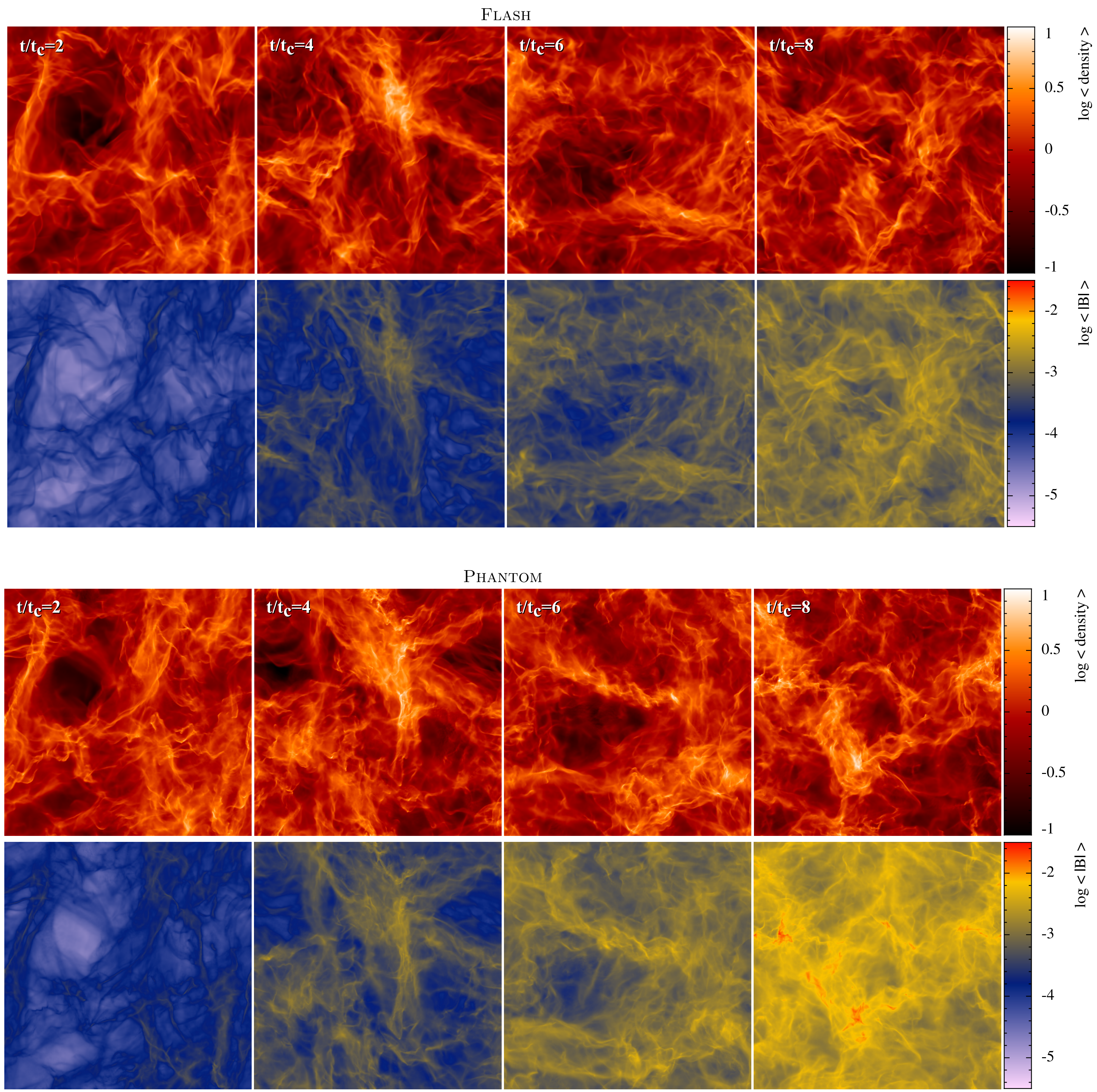
First core (100 x 100 au)



Second (protostellar) core (10 x 10 au)

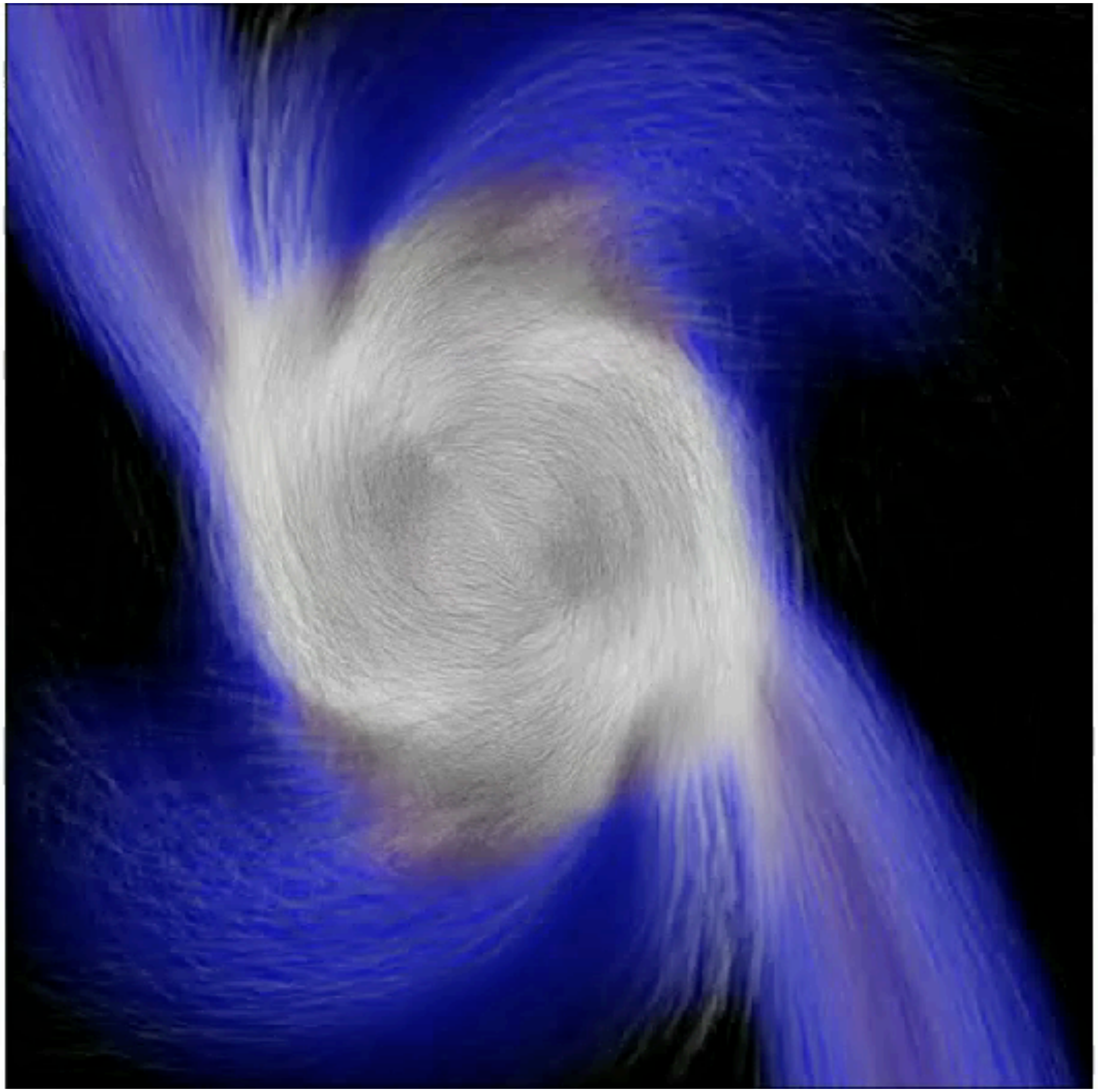
SMALL SCALE DYNAMO: FLASH VS PHANTOM

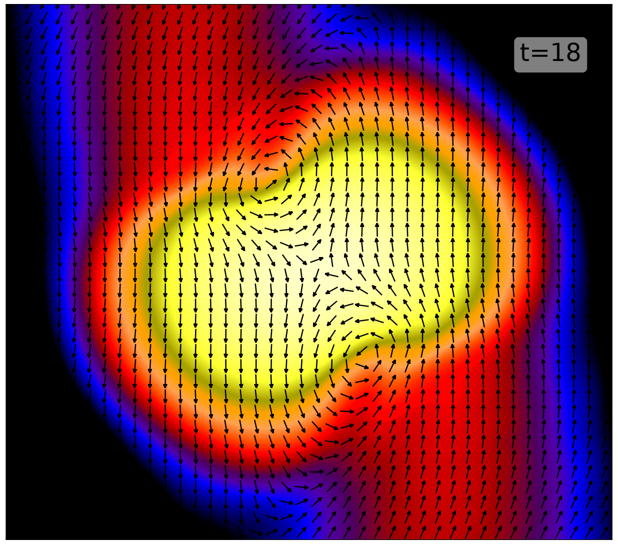
Tricco, Price & Federrath (2016)

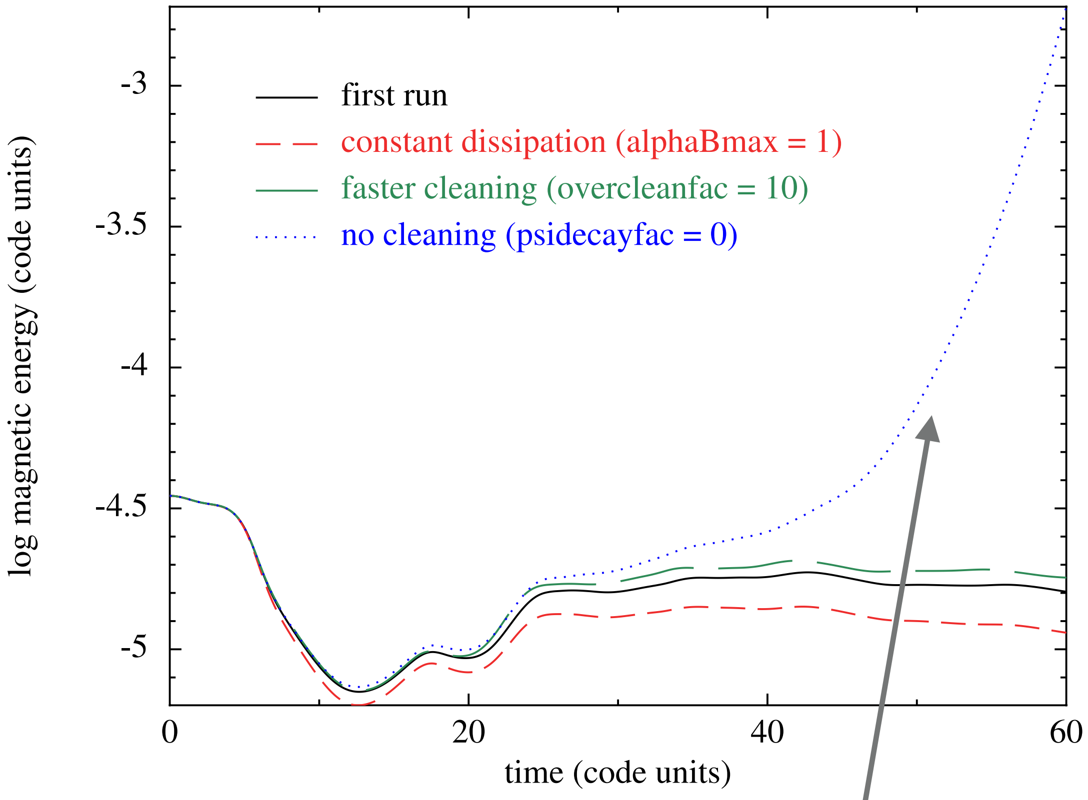


MAGNETIC FIELDS IN TIDAL DISRUPTION EVENTS

Bonnerot, Price, Rossi, Lodato (2017), submitted to MNRAS



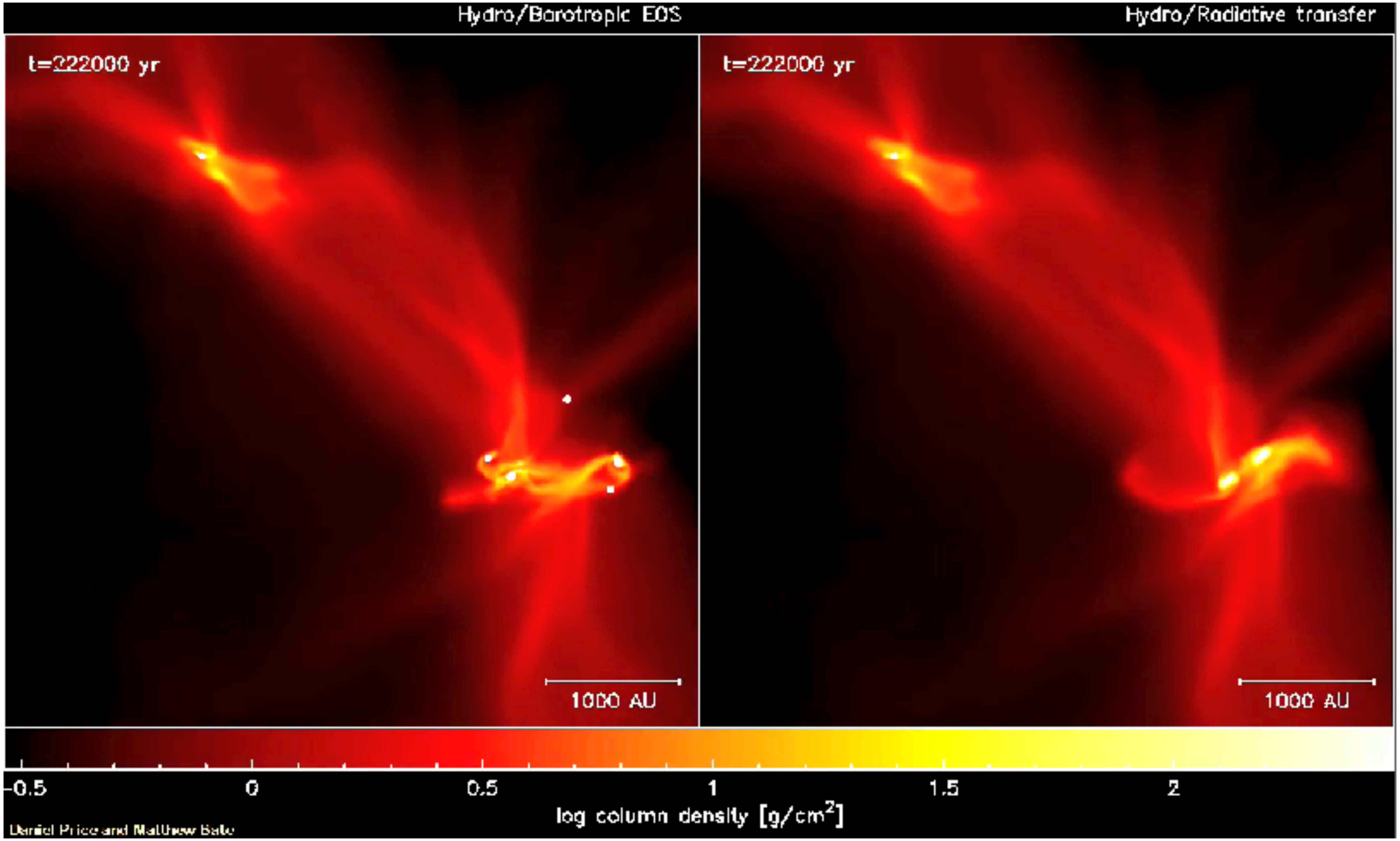




Danger! Can get artificial dynamo using "8 wave scheme" w/out div B cleaning

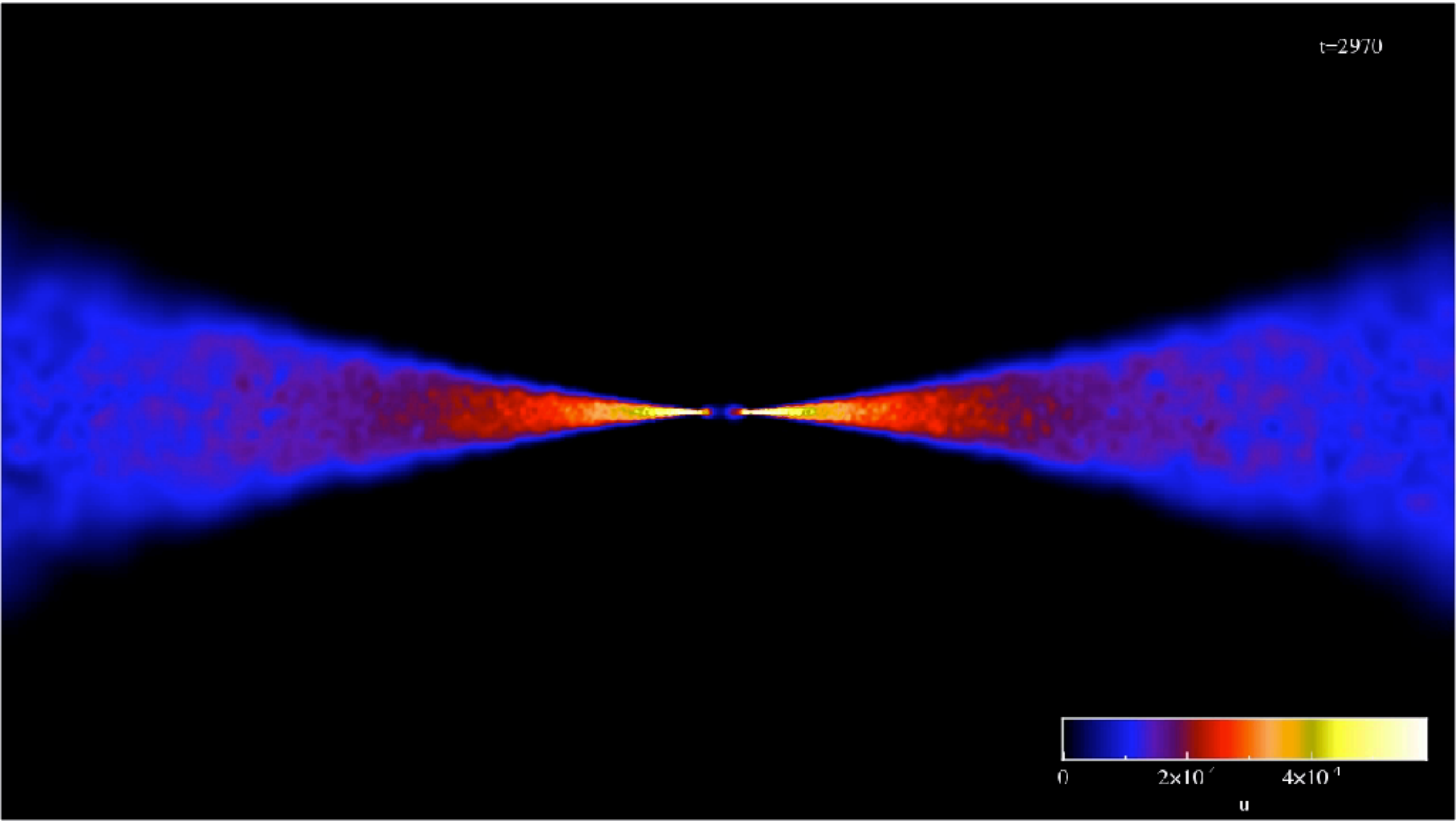
RADIATION

WHY RADIATION?



But Flux Limited Diffusion is both slow and wrong…

PHANTOM + MCFOST MONTE-CARLO RADIATION CODE

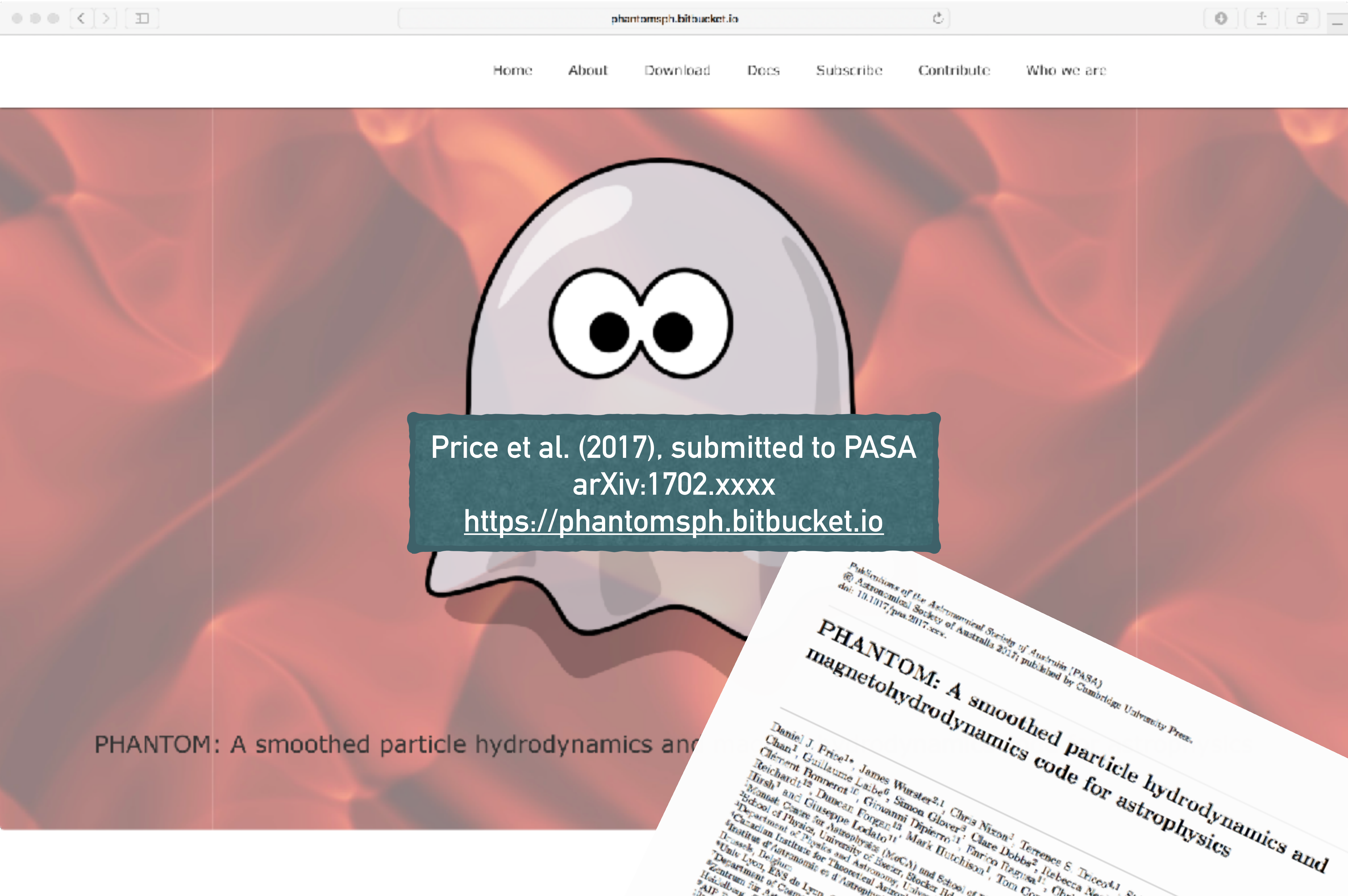


Mentiplay, Pinte & Price (2017), in prep.

SUMMARY

1. MHD in SPH is now fairly mature, useable out of the box for many practical applications
2. New one fluid dust method great for handling small grains / short stopping times
3. Direct coupling with Monte-Carlo radiation codes seems feasible, at least for disc studies

PHANTOM IS NOW PUBLIC



Price et al. (2017), submitted to PASA
arXiv:1702.xxxx
https://phantomsph.bitbucket.io