The code RoSSBi : a multi-fluid method for the formation of primordial bodies in protoplanetary disks

Carving through the Codes : Challenges in Computational Astrophysics

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Protoplanetary systems are composed of a proto-star, a gas disk (due to angular momentum) and solid components embeded (dust, pebbles, planetesimals, planets).

The level of complexity of the dynamics can be high : instabilities, accretion, self-gravity and spiral waves, etc...

In the lowest level of complexity, the system looks like as steady with :

- Spherical gravitational field (star),
- Axisymetric gas flow, with no radial accretion ($V_r = 0$)

This system is not necessary easy to evolve numerically !

I designed the code RoSSBi from scratch to address the dynamics of protoplanetary disks. The main properties are :

- 2D polar coordinates, or 3D spherical, to fit the main symmetry of the system.
- Finite Volume Method approach (FVM), on a fixed grid.
- At least second order in time and space.

I try to be as close as possible to the Euler equations of conservation :

$$\int_t^{t+\Delta t} \partial_t \int_{\mathcal{V}} \mathcal{E}uler \, d\mathcal{V} \, dt' = \int_t^{t+\Delta t} \oint_{\mathcal{S}} \mathcal{F}\vec{lux} \cdot d\vec{\mathcal{S}} \, dt' + \int_t^{t+\Delta t} \int_{\mathcal{V}} \mathcal{S}ource \, d\mathcal{V} \, dt' \tag{1}$$

To be second order in time, we need to approximate for example the sources as : $\mathcal{S}ource = \mathcal{S}ource(t) + t' \, \frac{\mathcal{S}ource(t+\Delta t/2) - \mathcal{S}ource(t)}{\Delta t/2}$

The problem of well-balanced scheme is due to the radial variation of the steady low complexity solution of the system, which can be the MMSN model. The radial velocity is null, and the azimuthal velocity of this solution is :

$$V_0^2(r) = r^2\Omega_K^2(r) + r\sigma_0^{-1}(r)\partial_r P_0(r)\,. \tag{2}$$

The steady axisymmetric Euler equations reduce to :

$$\frac{1}{r}\partial_r r P^* P_0(r) = \sigma^* V_\theta^{*\,2}\sigma_0(r)\frac{V_0^2(r)}{r} - \sigma^*\sigma_0(r)\frac{V_K^2(r)}{r} + P^*\frac{P_0(r)}{r}\,, \tag{3}$$

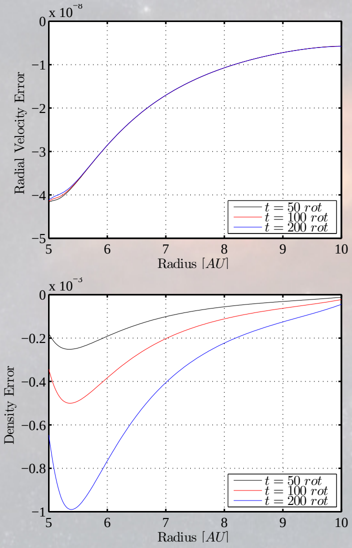
Integrating in space with the conservative form of the FVM gives :

$$\begin{aligned}\left[P^* P_0(r) r\right]_{r^-}^{r^+}\Delta\theta &= \sigma^* V_\theta^{*\,2}\int_{r^-}^{r^+}\sigma_0(r) V_0^2(r)\,dr\Delta\theta \\ &- \sigma^*\int_{r^-}^{r^+}\sigma_0(r) V_K^2(r)\,dr\Delta\theta + P^*\int_{r^-}^{r^+} P_0(r)\,dr\Delta\theta\end{aligned} \tag{4}$$

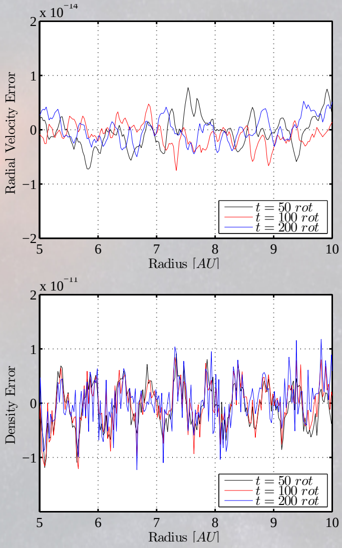
To obtain a well-balanced scheme, we use the following relations :

$$\begin{aligned}\int_{r^-}^{r^+} \sigma_0(r)\, V_0^2(r)\, dr\Delta\theta &= \int_{r^-}^{r^+} \sigma_0(r)\, V_K^2(r)\, dr\Delta\theta + \int_{r^-}^{r^+} r\partial_r P_0(r)\, dr\Delta\theta \\ &= [P_0(r) r]_{r^-}^{r^+} \Delta\theta \\ &+ \int_{r^-}^{r^+} \sigma_0(r)\, V_K^2(r)\, dr\Delta\theta - \int_{r^-}^{r^+} P_0(r)\, dr\Delta\theta \end{aligned} \tag{5}$$

The two last integrals can be approximated at second order, as long as the same values are used for the Keplerian and pressure source terms.

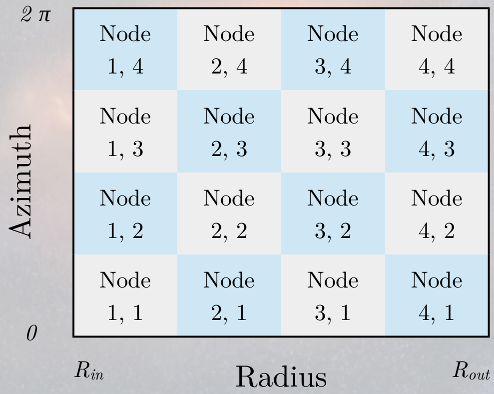


Unbalanced Scheme



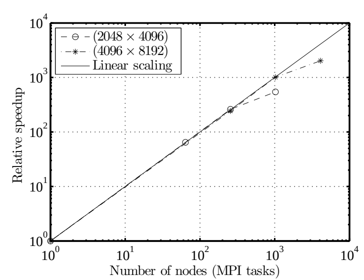
Balanced Scheme

We use OpenMP paralellism on local blocks, and MPI parallelism for the whole grid, to perform global 2D simulations.

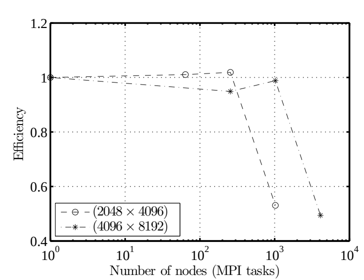


Production runs on the Piz Daint Swiss system scale linearly for blocks of 128×256 :

- up to 256 nodes (4096 threads) for $(N_r, N_\theta) = (2048, 4096)$
- up to 1024 nodes (16384 threads) for $(N_r, N_\theta) = (4096, 8192)$



Scaling as function of node numbers



Efficiency to linear scaling

Multi-fluid approach : the drag force problem

The motion of a solid grain embeded in protoplanetary disks is driven by different forces :

$$m\vec{a}(grain) = \vec{G}(star) + \vec{F}(friction) + \vec{F}(collisions) \tag{6}$$

- As most of the acceleration is compensed by the star gravity, the dominant force is the drag friction for small grains.
- As long as the distance of efficiency of the drag is shorter than the mean distance between grains, collisions are unlikely.
- The grains will have a **collective motion imposed by the gas**, and thus can be treated as a fluid.

The problem for coupled fluids, is when the friction is on short timescale ($\sim$ small grains).
Resolving the friction explicitely will impose timesteps shorter than for advection, and make the solution slow and inaccurate.

We consider the impulsion of the gas and particle fluids, $\vec{P}_g = \sigma_g \vec{V}_g$ and $\vec{P}_p = \sigma_p \vec{V}_p$, respectively, fully coupled by the drag force.

$$\partial_t \vec{P}_g = \vec{\mathcal{A}}_g + \Omega_k(r) S_t^{-1} \left(\vec{P}_p - \frac{\sigma_p}{\sigma_g} \vec{P}_g \right), \qquad (7)$$

$$\partial_t \vec{P}_p = \vec{\mathcal{A}}_p - \Omega_k(r) S_t^{-1} \left(\vec{P}_p - \frac{\sigma_p}{\sigma_g} \vec{P}_g \right). \qquad (8)$$

If we introduce $\Delta\vec{\mathcal{A}} = \vec{\mathcal{A}}_p - \epsilon \vec{\mathcal{A}}_g$, $\Delta\vec{P} = \vec{P}_p - \epsilon \vec{P}_g$, and the drag frequency $\omega_p = \Omega_k(r) S_t^{-1}$, one obtains

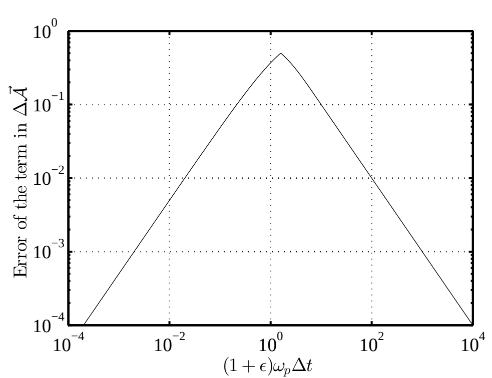
$$\partial_t \Delta\vec{P} = \Delta\vec{\mathcal{A}} - (1+\epsilon)\, \omega_p \Delta\vec{P}. \qquad (9)$$

A solution over the timestep, $t' \in [t,\ t + \Delta t]$ is

$$\begin{aligned} \Delta\vec{P}(t') &= \Delta\vec{P}(t) \exp[-(1+\epsilon)\omega_p t'] \\ &+ \frac{\Delta\vec{\mathcal{A}}(t)}{(1+\epsilon)\omega_p} \left(1 - \exp[-(1+\epsilon)\omega_p t'] \right), \end{aligned} \qquad (10)$$

To keep the second order accuracy of the advection scheme and not couple advection with friction, we can get rid of the $\Delta\vec{\mathcal{A}}$ term because :

- *(i)* $(1+\epsilon)\omega_p\Delta t < 1$, third order term (superposition of solutions)
- *(ii)* $(1+\epsilon)\omega_p\Delta t > 1$, short friction timescale limit



Then the time integration of the motion equations becomes :

$$\vec{P}_g(t+\Delta t) - \vec{P}_g(t) = \int_t^{t+\Delta t} \vec{\mathcal{A}}_g\,dt' + \int_0^{\Delta t} \omega_p \Delta\vec{P}(t)\exp[-(1+\epsilon)\omega_p t']\,dt',$$
$$\vec{P}_p(t+\Delta t) - \vec{P}_p(t) = \int_t^{t+\Delta t} \vec{\mathcal{A}}_p\,dt' - \int_0^{\Delta t} \omega_p \Delta\vec{P}(t)\exp[-(1+\epsilon)\omega_p t']\,dt', \qquad (11)$$

We can thus compute implicitely the time integral of the friction, and obtain :

$$\int_0^{\Delta t} \omega_p \Delta\vec{P}(t)\exp[-(1+\epsilon)\omega_p t']\,dt' = \frac{\Delta\vec{P}(t)}{(1+\epsilon)} \times \left[1 - \exp[-(1+\epsilon)\omega_p \Delta t]\right]. \qquad (12)$$

Keeping in mind **the integral form of the FVM** helps to find accurate solutions of the equations for critical conditions.

The method can be expanded to two dust fluids, and we obtain :

$$\partial_t \vec{P}_g = +\omega_{p1}\left(\vec{P}_{p1} - \epsilon_1 \vec{P}_g\right), \quad (13)$$
$$+\omega_{p2}\left(\vec{P}_{p2} - \epsilon_2 \vec{P}_g\right) \quad (14)$$
$$\partial_t \vec{P}_{p1} = -\omega_{p1}\left(\vec{P}_{p1} - \epsilon_1 \vec{P}_g\right) \quad (15)$$
$$\partial_t \vec{P}_{p2} = -\omega_{p2}\left(\vec{P}_{p2} - \epsilon_2 \vec{P}_g\right) \quad (16)$$
$$(17)$$

In the same notation system as before we obtain after replacing the gas equation :

$$\partial_t \Delta\vec{P}_{p1} = -\left(1+\epsilon_1\right)\omega_{p1}\Delta\vec{P}_{p1} - \epsilon_1\omega_{p2}\Delta\vec{P}_{p2} \quad (18)$$
$$\partial_t \Delta\vec{P}_{p2} = -\epsilon_2\omega_{p1}\Delta\vec{P}_{p1} - \left(1+\epsilon_2\right)\omega_{p2}\Delta\vec{P}_{p2}. \quad (19)$$

The solution of the system is obtained with the exponential of the matrix defining the linear system of differential equations.
We have implemented the 2 dust fluids case, and are working on the 3 dust fluids solution (exponential of a 3×3 matrix...)

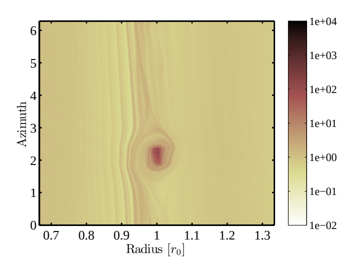
The following results are taken from :

- Surville, C., Mayer, L., Lin, D. N. C. "Dust capture and long-lived density enhancements triggered by vortices in 2D protoplanetary disks" 2016, *The Astrophysical Journal*, 831, 82
- Surville, C., and Mayer, L., "Effect of small grains on the evolution of vortices in 2D PPDisks" 2017, *The Astrophysical Journal*, under review
- Surville, C., Mayer, L., and Alibert, Y., "Dust rings triggered by super Earths" 2017, *The Astrophysical Journal*, in preparation

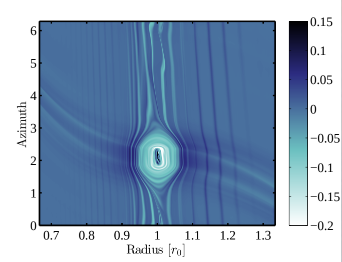
Examples of applications

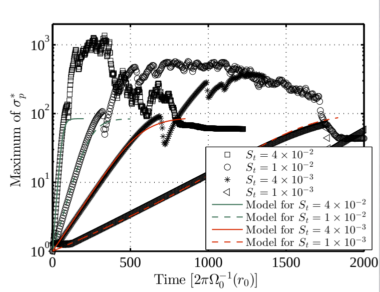
Dust capture in vortices and ring formation

Dust density

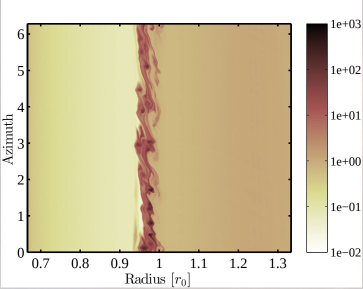


Rossby number

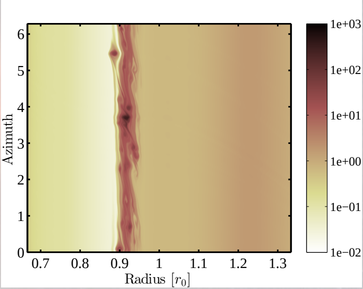




- Capture of solids inside the vortex creates an exponential growth of the dust density in the vortex.
- The linear model fits the numerical results, even for frictions in the critical regime $(1+\epsilon)\omega_p \Delta t \sim 1$



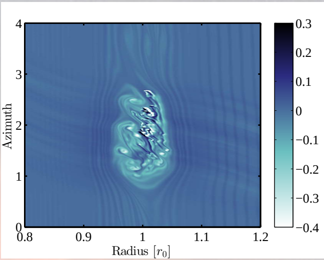
Dust density for $S_t = 4 \times 10^{-2}$



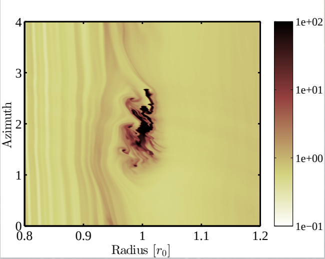
Dust density for $S_t = 1 \times 10^{-3}$

Larger grains tend to produce norrow rings with dense eddies, while smaller grains produce wide rings with smoother structures (resolution issue ?).

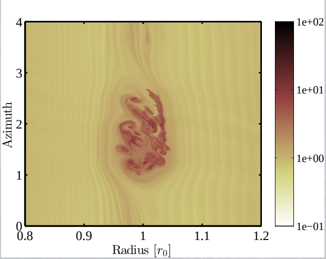
The structure of dust rings produced by vortices are different as function of the grain size



Rossby number of the gas

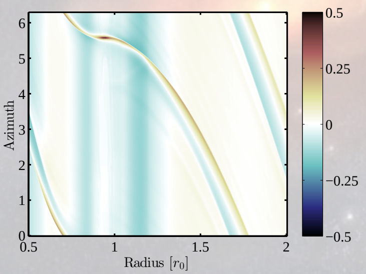


Density of large grains ($S_t = 0.25$)

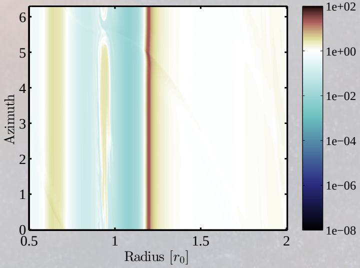


Density of small grains ($S_t = 0.025$)

We performed a study with Yann Alibert of a typical setup of a 20 Earth masses planet orbiting at 5 AU with dust of $S_t = 10^{-1}$ and $S_t = 5 \times 10^{-2}$ (3.2 and 1.6 cm)
We observe gap opening and accumulation of solids at the outer edge.

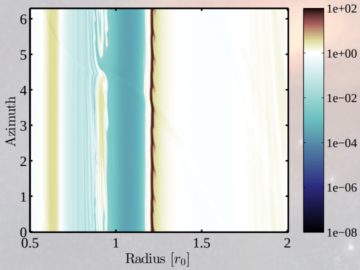


Gas density at $t = 300$ rot

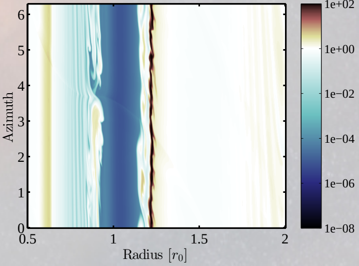


Dust density for $S_t = 5 \times 10^{-2}$

The edge becomes unstable to the drag/vorticity instability (in competition with the RWI)

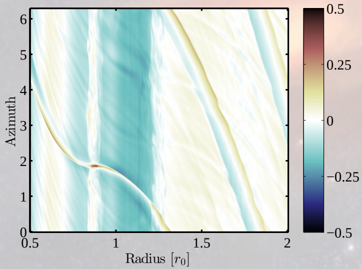


Dust density at $t = 400$ *rot*

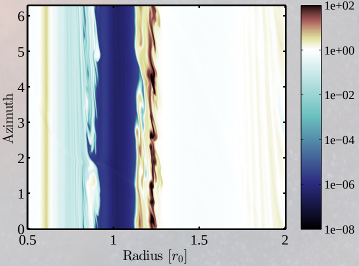


Dust density at $t = 500$ *rot*

As time goes, and as the planet migrates, the gap widens, and the dust ring forms with eddies of dust-to-gas ratio larger than unity.

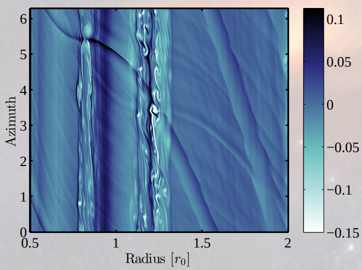


Gas density at $t = 580$ *rot*

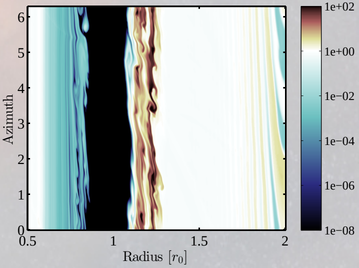


Dust density for $S_t = 5 \times 10^{-2}$

Second ring formation : the gap moves with the planet, triggering a second instability and ring formation.

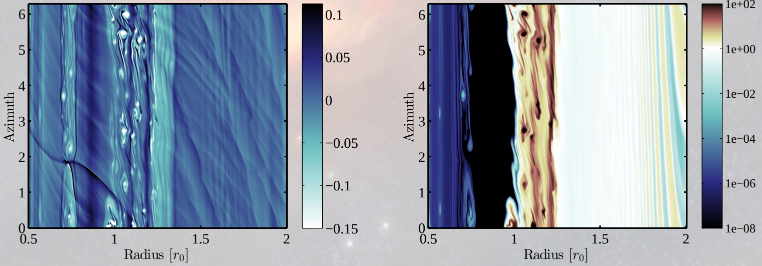


Gas vorticity at $t = 800$ *rot*



Dust density for $S_t = 5 \times 10^{-2}$

The process continues and additional dust rings can form, which are decorrelated from the initial gap.

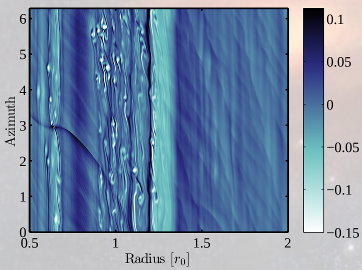


Gas vorticity at $t = 1200$ *rot*   Dust density for $S_t = 5 \times 10^{-2}$

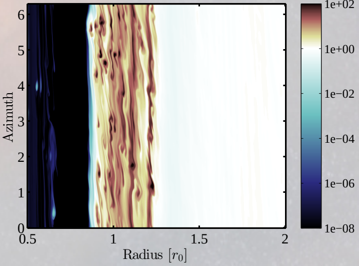
Examples of applications

Planet-disk interaction : dust ring formation

The planet has cleaned the solid component inside the gap. But a wide dusty region with rings and eddies is left in the outer parts.



Gas vorticity at $t = 1600\ rot$



Dust density for $S_t = 5 \times 10^{-2}$

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- The high resolution achievable with multi-fluids (2 dust populations) is an innovation to understand the dynamics of primordial small size solids.

Conclusions

- The code RoSSBi is designed to solve the complexity of protoplanetary disks dynamics thanks to a high order FFM and a well balanced scheme.
- To stay close to the conservative form of the equations helps to find implicit or accurate solutions of the source terms.
- The high resolution achievable with multi-fluids (2 dust populations) is an innovation to understand the dynamics of primordial small size solids.
- We discovered a new drag instability, *the drag/vorticity instability*, that destroys vortices, but sustains dust rings under many conditions.

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- We discovered a new drag instability, *the drag/vorticity instability*, that destroys vortices, but sustains dust rings under many conditions.
- (Close-)Future improvements, like disk self-gravity, 3D spherical, will help to understand the complex dynamics of solids in PPDisks, and to upgrade planet formation models, in particular the core accretion scenario.

Thank you for your attention !